11.7 teacher notes

Objective

- Divide polynomials using long division.

Vocabulary

Dividend
Divisor
Descending order
Partial dividend
Remainder
Quotient (Lesson 1-2)

Prerequisites

Dividing numbers using long division

Multiplying a monomial by a polynomial

Adding and subtracting polynomials

Writing polynomials in order of decreasing degree by applying the Commutative Property of Addition

Get Started

- Write the equation $513 \div 22 = ?$ on the board and ask students to find the answer. $\left(23 \text{ remainder } 7 \text{ or } 23\frac{7}{22}\right)$. Remind them they will have to use long division. Give the class several minutes to complete the division.
- Say, “Now, let’s review the process you used.” Ask students to describe their methods. Write the heading “Steps to Long Division” on the board.
- Students may have difficulty expressing the exact method. Use the opportunity to review the vocabulary of long division. Guide them to say that their first step was to divide 22 (the *divisor*) into 51 (a *partial dividend*). Write “1. Divide” on the board under the heading. Since 22 goes into 51 at most two times, a two was placed in the tens place of the quotient.
- Next, they multiplied two (the partial quotient) by 22 (the divisor). Write “2. Multiply” on the board.
- Next, they subtracted the product, 44, from 51. Write “3. Subtract” on the board.

- Next, they brought down the three to make a partial dividend of 73. Write "4. Bring down" on the board.
- Ask the class to continue the description of the solution and point out that the steps "Divide, Multiply, Subtract, Bring down" are repeated until the partial dividend is less than the divisor. Remind the class the remainder (seven) can be expressed as a fraction and added to the quotient.
- Say, "Today, we will use the same process we used here to divide polynomials using long division. We will follow the steps "Divide, Multiply, Subtract, Bring Down, Repeat" to divide one polynomial by another.

Section 1

Expand Their Horizons

In this lesson, students will be shown how to use long division to divide polynomials. The long division algorithm should be familiar to students because they have used the process to find numerical quotients. The method used to divide polynomials is identical to the method used to divide numbers.

Before starting the lesson, consider reviewing how to subtract polynomials. When subtraction of polynomials is done in vertical format, like terms should be aligned or "stacked" vertically. Subtraction is the same as addition of the opposite. So, to find the difference, find the opposite of the bottom polynomial (by changing the sign of each of its terms) and then, add the polynomials by combining like terms. Because like terms are aligned vertically, simply add the terms in each column to find the sum.

$$\begin{array}{rrrrr} & 3b^3 & & -4b & +2 \\ - & & (6b^2 & -b & -7) \\ \hline \end{array} \qquad \text{or}$$

$$\begin{array}{rrrrr} & 3b^3 & & -4b & +2 \\ + & & -6b^2 & +b & +7 \\ \hline & 3b^3 & -6b^2 & -3b & +9 \end{array}$$

Before dividing a polynomial by a polynomial, it is important the dividend and the divisor each be written in descending order. A polynomial in one variable is written in descending order when the power of the variable decreases from left to right. The polynomial $-2 + 3x^2 - 4x$ can be written in descending order as $3x^2 - 4x - 2$. Notice, the Commutative Property of Addition is used to rearrange the terms without changing the value of the expression. Both the divisor and dividend should have no missing powers of the variable. This arrangement is to help ensure proper alignment of like terms in the division workspace. If a polynomial has missing powers of the variable, the situation can be remedied by adding zero-value terms. For example, the polynomial $-4p^3 - 4$ can be written in this form by adding the terms $0p^2$ and $0p$ so that the expression is $-4p^3 + 0p^2 + 0p - 4$. Notice, adding terms whose coefficients are zero does not change the value of the expression: $-4p^3 + 0p^2 + 0p - 4 = -4p^3 + 0 + 0 - 4 = -4p^3 - 4$.

To divide polynomials using long division, a four-step process is used. Consider the expression $(6x^2 + 11x + 1) \div (3x + 1)$. In each iteration (one cycle of the four-step process), the first term of the dividend (or partial dividend) is divided by the first term of the divisor: $\frac{6x^2}{3x} = 2x$. The result, $2x$, is placed above the first term of the dividend. Next, the result is multiplied by the entire divisor, and the product is written under the dividend (or partial dividend) so that like terms are aligned.

$$\begin{array}{r} 2x \\ 3x+1\overline{)6x^2+11x+1} \\ \underline{6x^2+2x} \end{array}$$

The product is then subtracted from the dividend (or partial dividend). The left most term in the difference should always be zero.

$$\begin{array}{r} 6x^2 + 11x \\ -\ (6x^2 + 2x) \\ \hline 0 + 9x \end{array}$$

Finally, the next term (if any) of the dividend is brought down: one. The process is repeated using the partial dividend formed by the difference and the brought-down term: $9x + 1$. Iterations are performed until the degree of the difference is less than the degree of the divisor. The remainder, which consists of difference and any additional terms in the dividend, is expressed as a fraction and added to the quotient.

$$\begin{array}{r} 2x + 3 \phantom{{}+1} \\ 3x + 1 \overline{\smash{)}\, 6x^2 + 11x + 1} \\ -(6x^2 + 2x) \phantom{{}+1} \\ \hline 9x + 1 \\ -9x + 3 \\ \hline -2 \end{array}$$

So, $(6x^2 + 11x + 1) \div (3x + 1) = 2x + 3 - \frac{2}{3x + 1}$.

As another example, consider the expression $(x^4 + 2x^3 + x + 1) \div (x^3 + x^2)$.

$$\begin{array}{r} x + 1 \phantom{{}+0x^2+x+1} \\ x^3 + x^2 \overline{\smash{)}\, x^4 + 2x^3 + 0x^2 + x + 1} \\ -(x^4 + x^3) \phantom{{}+0x^2+x+1} \\ \hline x^3 + 0x^2 \phantom{{}+x+1} \\ -(x^3 + x^2) \phantom{{}+x+1} \\ \hline -x^2 + x + 1 \end{array}$$

Here, the process is continued until the degree of the remainder is less than the degree of divisor (the degree of $-x^2$ is less than the degree of $x^3 + x^2$). The additional terms of the dividend, $x + 1$, are brought down and used as the numerator of the remainder term. So, $(x^4 + 2x^3 + x + 1) \div (x^3 + x^2) = x + 1 + \frac{-x^2 + x + 1}{x^3 + x^2}$.

1 **Divide: $(x^2 + 3x - 18) \div (x + 6)$.** This problem requires no preparation before beginning the long division process, because both dividend and divisor are written in decreasing degree of x and have no missing powers of x. Begin by dividing x into x^2. Write the result, x, in the quotient. Next, multiply x by the divisor, $x + 6$. Write the result, $x^2 + 6x$, below the partial dividend and subtract. The difference is $-3x$. Bring down -18 to form the partial dividend: $-3x - 18$. Repeat this process. Divide x into $-3x$, writing the result, -3, in the quotient. Multiply -3 by $x + 6$, and write the result, $-3x - 18$, below the partial dividend, aligning like terms. Subtract. The difference is zero. This problem has no remainder.

Point out to the class they should check their answers to long division problems using the formula (Quotient)(Divisor) + (Remainder) = (Dividend). If the problem has no remainder, (Quotient)(Divisor) = (Dividend). So, they can check their solutions to "Guided Notes Question 1" by testing whether the equation $(x - 3)(x + 6) = x^2 + 3x - 18$; this is true.

Common Error Alert

Students may stumble in the second iteration of the process when they must multiply the expression in the quotient by the divisor. They are often tempted to incorrectly multiply the *entire* expression in the quotient by the divisor. For example, in "Guided Notes Question 1," they may multiply $x - 3$ by $x + 6$ instead of multiplying -3 by $x + 6$. Remind them that only the most recent term in the quotient is multiplied. If multiplying the entire expression is a continual problem for students, advise them to develop the habit of covering the "older" terms of the quotient with their finger.

2 **Divide: $(8 + 4x^2) \div (2x + 1)$.** The dividend is not written in decreasing degree of x, and it has a missing power of x. Before dividing, change the dividend $8 + 4x^2$ into the equivalent expression $4x^2 + 0x + 8$. Begin by dividing $2x$ into $4x^2$. Write the result, $2x$, in the quotient; then, multiply

$2x$ by the entire divisor. Write the result, $4x^2 + 2x$, below the quotient and subtract. Bring down + 8 to form the partial dividend $-2x + 8$. Repeat this process. Divide $2x$ into $-2x$. Write the result, -1, in the quotient and multiply by $2x + 1$. Write the result, $-2x - 1$, below the partial dividend and subtract. The difference is nine. Since the degree of nine is zero (all constants have degree zero), and the degree of the divisor is one (the highest power of x in $2x + 1$ is one), the process stops. Nine is the remainder. The answer is $2x - 1 + \frac{9}{2x + 1}$.

Review the vocabulary associated with the answer to a division problem. The *quotient* is the polynomial part of the answer; therefore, in "Guided Notes Question 2," the quotient is $2x - 1$. The remainder is nine. Similarly, in the numerical division problem $11 \div 2$, the quotient is five, and the remainder is one.

Look Beyond

The type of repetitive process described in this lesson is sometimes referred to as an algorithm. An algorithm consists of a finite number of steps describing how the process should be performed. Algorithms, which require iteration, appear throughout mathematics. In such a process, the output for each iteration is used as the input for the next iteration. In long division, the partial dividend created by each iteration is used in the next iteration of the algorithm. Algorithm design and development have many applications in mathematics and computer science.

Additional Examples

1. Divide: $(t^2 - 4t + 3) \div (3 + t)$.

Write the divisor in order of decreasing degree before performing the calculation.

$$\begin{array}{r}
t - 7 \\
t + 3\overline{)\ t^2 - 4t + 3} \\
-(t^2 + 3t) \\
\hline
-7t + 3 \\
-(-7t - 21) \\
\hline
24
\end{array}$$

$$(t^2 - 4t + 3) \div (3 + t) = t - 7 + \frac{24}{t + 3}$$

2. Divide: $(6x^3 - 5x^2 + 15x - 5) \div (2x^2 - x + 3)$.

In this case, the divisor is a trinomial. Use the same method described in this lesson.

$$\begin{array}{r}
3x - 1 \\
2x^2 - x + 3\overline{)\ 6x^3 - 5x^2 + 15x - 5} \\
-(+6x^3 - 3x^2 + 9x) \\
\hline
-2x^2 + 6x - 5 \\
-(-2x^2 + x - 3) \\
\hline
5x - 2
\end{array}$$

$$(6x^3 - 5x^2 + 15x - 5) \div (2x^2 - x + 3) = 3x - 1 + \frac{5x - 2}{2x^2 - x + 3}$$