Geometry

Module 9

Characteristics of Geometric Shapes

Lesson 5
Inductive and Deductive Reasoning

Teacher Notes 9.5

Objectives

- Define and apply deductive reasoning to solve problems involving geometric relationships.
- Define and apply inductive reasoning to solve problems involving number patterns and geometric relationships.

Prerequisites

Defining, classifying, and using the properties of angles and polygons

Identifying and defining parts of a circle

Vocabulary

Inductive reasoning
Deductive reasoning
Conjecture
Vertex of a triangle (8.4)
Polygon (9.1)
Diagonal
Counterexample
Line segment (8.1)
Quadrilateral (9.1)
Vertical angles (8.3)
Supplementary (8.3)
Acute triangle (8.4)
Equiangular triangle (8.4)
Equilateral triangle (8.4)
Obtuse triangle (8.4)
Straight angle (8.2)
Chord (9.3)

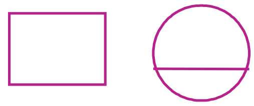
Get Started

- Write the following statements on the board and have students determine which of the statements are true and which are false:

  All quadrilaterals are trapezoids. **False**
  All parallelograms are quadrilaterals. **True**
  All triangles are polygons. **True**
  All chords are diameters. **False**

- Have students explain how they determined which statements were true and which were false. Elicit that sometimes it is easiest to determine the truth of a statement by trying to think of a case where it is not true.

- Have volunteers come to the board to draw figures that show why the two false statements are false.

Possible answers: 

Subtopic 1 — **Inductive Reasoning**

Expand Their Horizons

In this subtopic, students learn about inductive reasoning. Conclusions based on inductive reasoning are conjectures, or educated guesses. This is because inductive reasoning, unlike deductive reasoning, is based on observation. It looks at specific examples and uses those examples to draw a general conclusion. Inductive reasoning includes experimentation, such as measuring the angles of several triangles to conjecture that the sum of the angles of any triangle is 180°.

Inductive reasoning also involves pattern discovery. For numerical problems, it often helps to keep track of observations in a table. The Galactic Chess Tournament problem solved in the lesson is the classic handshake problem introduced in probability and statistics classes. The handshake problem asks: *"In a room of n people, how many handshakes are possible if each person shakes hands with everyone else in the room?"* $\left(\frac{n(n-1)}{2}\right)$

It takes one counterexample to prove a conjecture false. For instance, given the expression $\frac{1}{x}$, one might make the conjecture that for any value of x, the expression is less than or equal to one. A counterexample to that conjecture is $x = \frac{1}{3}$ because $1 \div \frac{1}{3} = 3$.

Common Error Alert:

Students may assume that since one counterexample can prove a statement false, then one example can prove a statement true. To show the error in this line of thinking, write the statement: *"All right triangles are isosceles triangles"* on the board and ask students to draw a right isosceles triangle. Discuss with students whether this is proof of the statement and lead students to see that they can also draw a right triangle that is not isosceles.

It is important for students to understand that observation is not proof. Consider the sequence 2, 4, 6, 8, 10 .... Each number in the pattern increases by two, and one can form the conjecture that the pattern is to "add two" each time. However, the pattern could be "add two four times, then add 3,000."

Although inductive reasoning is not fool-proof, it still plays a very important role in mathematics. It provides mathematicians, as well as students of mathematics, with a place to start. By studying examples and patterns, they make and test conjectures. When no counterexamples are found, they can then test their theories using deductive reasoning, which is studied in the next subtopic.

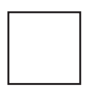
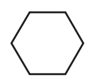
1 Ask if there is a quadrilateral that does not have four right angles. A counterexample is any four-sided polygon such that all four angles are not right angles.

2 Look for a pattern. The number of equal segments in each circle increases by 2: 2, 4, 6, 8, and 10.

Students might also find patterns with the number of lines segments and angle measures.

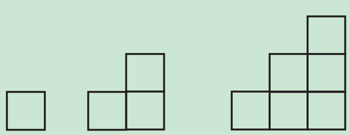
3 There are two patterns. First, the number of sides in each polygon increases by one. Second, the name of each figure alternates with the figure itself.

Here, the lesson reminds students that our answers are conjectures based on a pattern that we see. The correct pattern may be different, especially if we have not been given enough terms to find it. For all we know, the sequence might continue as:

Triangle,  , Pentagon,  , 7, Octagon, ...

Additional Examples

1. Find the next two terms in the pattern shown below.



2. Make a conjecture about the relationship between the number of sides of a polygon and the sum of the measures of the angles.

Polygon	# of Sides	Sum of Angle Measures
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°

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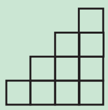
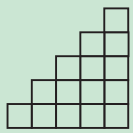
The second term has two more squares than the first. The third term has three more squares than the second. The fourth term will have four more squares than the third and the fifth will have five more than the fourth.

The next two terms are:





Each measure is a multiple of 180°. The first is $1 \times 180°$, the second is $2 \times 180°$, the third is $3 \times 180°$, and the fourth is $4 \times 180°$.

Notice that in each, the first factor is two less than the number of sides in the polygon.

The sum of the angles of a polygon is $(n - 2)180°$, where n is the number of sides.

Subtopic 2 — Deductive Reasoning

Expand Their Horizons

In this subtopic, students are introduced to deductive reasoning. Deductive reasoning moves in the opposite direction of inductive reasoning in that it draws specific conclusions from general facts. Deductive reasoning, unlike inductive reasoning, is considered proof because it uses accepted definitions and rules to make a conclusion.

Study the differences between the inductive and deductive reasoning below.

Inductive reasoning
Given: $1 \times 6 = 6$, $1 \times 5 = 5$, $1 \times 10 = 10$.
Conjecture: The product of one and any natural number is the natural number.

Deductive reasoning
Given: The product of one and any natural number is the natural number.
Conclusion: $1 \times 8 = 8$.

In the lesson, students are introduced to the idea of proof as the characters prove that vertical angles are congruent. Reinforce the difference between inductive and deductive reasoning by explaining that measuring hundreds of vertical angles and by showing that each pair is congruent would be inductive reasoning, but by using given facts and logical thinking to progress from one step to another is deductive reasoning.

4 All equilateral triangles are also equiangular. The sum of the measures of the angles of a triangle is 180° and $180° \div 3 = 60°$. Therefore, all angles of an equilateral triangle measure 60°. An acute angle measures less than 90°, and an acute triangle has three acute angles. So, all equilateral triangles are acute triangles.

5 Shawn used inductive reasoning because he made a general conclusion based on specific examples. His conclusion was false because a diameter was a chord which passed through the center of a circle.

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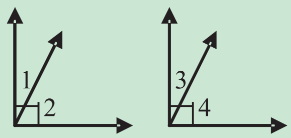
This was a case of invalid reasoning. Although the first statement was true, the second statement was not because every triangle has at least two acute angles. Any acute or right triangle made a sufficient counterexample.

Additional Examples

1. Use deductive reasoning to prove that a rectangle is a parallelogram.

A parallelogram has two pairs of parallel sides. Since a rectangle has two pairs of parallel sides, it is a parallelogram.

2. In the figures below, $m\angle 1 = m\angle 3$. Use deductive reasoning to prove that $m\angle 2 = m\angle 4$.



Each pair of angles is complementary:
$m\angle 1 + m\angle 2 = 90°$.
$m\angle 3 + m\angle 4 = 90°$.
Because both angle pairs equal 90°, they equal each other.
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$.
Because we are given that $m\angle 1 = m\angle 3$, replace $m\angle 1$ with $m\angle 3$ in the equation above: $m\angle 3 + m\angle 2 = m\angle 3 + m\angle 4$.
Subtract $m\angle 3$ from each side.
$m\angle 2 = m\angle 4$.

Look Beyond

Euclidean geometry is based on deductive reasoning. In high school geometry, students will study the geometry of Euclid and will learn methods of formal proof including paragraph proofs and two-column proofs. They will also study indirect proof, which assumes that what is to be proven is false. This leads to a contradiction, meaning that the assumption is false, and the original statement is true.

Connections

Advertisers use both inductive and deductive reasoning to sway people to buy their products. Many use invalid reasoning and play with people's emotions leading them to believe things such as, if the consumer does not buy our product, they are a bad mom/dad/pet owner.

Lawyers rely on their reasoning abilities to convince a jury that their client is either guilty or not guilty. They must also be able to identify and to explain invalid reasoning given by others.