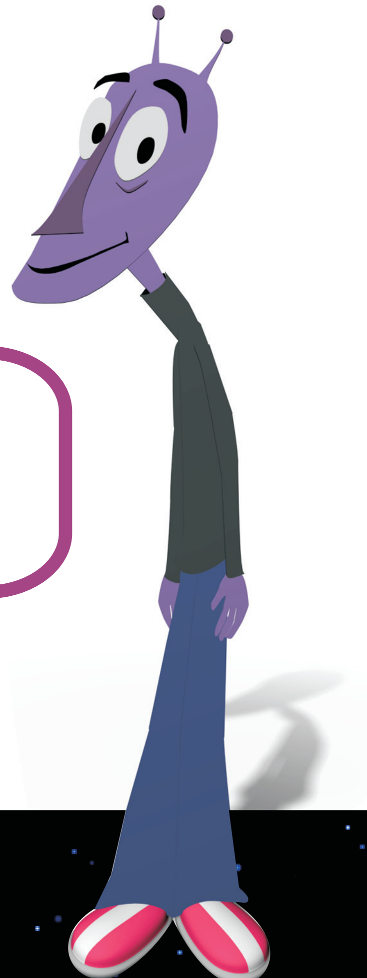
Geometry

Module 9

Characteristics of Geometric Shapes



Lesson 4
Similar Polygons

Teacher Notes 9.4

Objectives

- Identify shapes that have similarity.
- Identify similar figures and explore their properties.
- Develop the properties of similar figures (ratio of sides and congruent angles).
- Apply proportional reasoning to solve problems involving congruent or similar shapes (e.g., create scale drawings).

Power Talk Send

Prerequisites

Writing and simplifying ratios

Writing and solving proportions, including the percent proportion

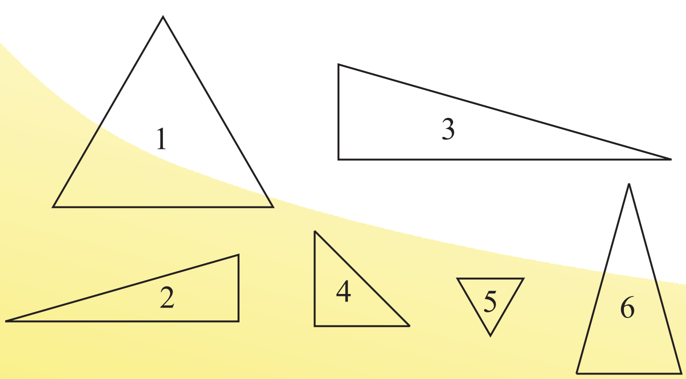
Finding the percent of a number

Vocabulary

Similar polygons
Polygon (9.1)
Triangle (8.4)
Similar triangles (8.6)
Congruent figures (8.5)
Proportion (7.2)
Ratio (4.1)
Rectangle (9.2)
Cross products (7.2)
Trapezoid (9.2)
Parallelogram (9.2)
Enlargement
Reduction
Scale factor
Percent proportion (7.5)
Scale
Scale drawing

Get Started

Review similar triangles by drawing the following on the board and by having students pick the triangles that appear similar. **(1 and 5; 2 and 3)**



- Have a volunteer come to the board and draw a triangle. Then, have other volunteers come up and draw triangles similar to the one drawn. Encourage them to draw similar triangles in different orientations. Choose two of the triangles and label the vertices. List the corresponding parts.

Subtopic 1 — Similar Polygons

Expand Their Horizons

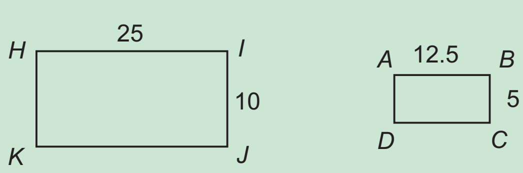
In this subtopic, students revisit similar triangles and are introduced to other similar figures including rectangles, trapezoids, and parallelograms. Similar figures are the same shape but not necessarily the same size. If they are the same size, the similar figures are also congruent.

To prove that two polygons which are not triangles are similar, teachers must show that the corresponding angles are congruent AND that the corresponding sides are proportional. For triangles, only one of the two statements had to be shown true. The Challenge Problems ask students to explain why this is the case for quadrilaterals. Squares and rectangles that are not squares both have right angles, but the sides are not in proportion, resulting in different shapes. Rectangles and parallelograms can have proportional side lengths but different angle measures.

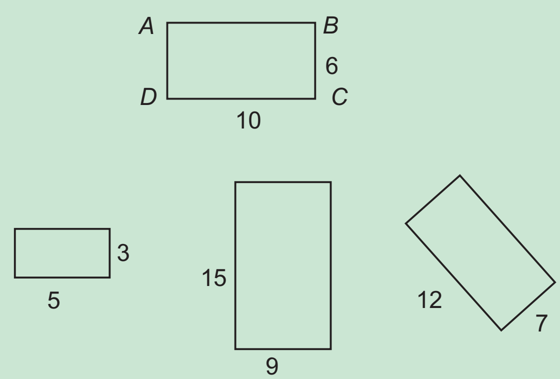
1 All the angles are right angles, so all corresponding angles are congruent. Form ratios with the corresponding sides. In $\frac{10}{7} = \frac{30}{20}$, the cross products are not equal. The sides are not in proportion. The rectangles are not similar.

Additional Examples

1. Is rectangle *HIJK* similar to rectangle *ABCD*? Explain.



2. Which rectangles are similar to rectangle *ABCD*?



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Since both figures are rectangles, the corresponding angles are all congruent right angles.

Check if the sides are proportional.

$$\frac{25}{12.5} = \frac{10}{5}$$

$$25 \times 5 = 12.5 \times 10$$

$$125 = 125$$

The cross products are equal. The equation is a proportion, and the rectangles are similar.

Since all the figures are rectangles, the corresponding angles are all congruent right angles. Check if the sides are proportional.

For the 5 × 3 rectangle:

$$\frac{6}{3} = \frac{10}{5}$$

$$30 = 30$$

Yes

For the 15 × 9 rectangle:

$$\frac{6}{9} = \frac{10}{15}$$

$$90 = 90$$

Yes

For the 12 × 7 rectangle:

$$\frac{6}{7} = \frac{10}{12}$$

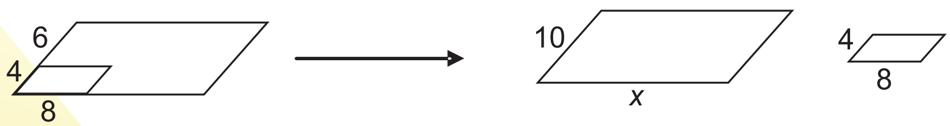
$$72 \neq 70$$

No

Subtopic 2 Finding Unknown Lengths

Expand Their Horizons

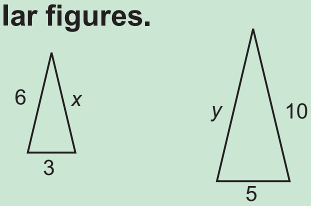
In this subtopic, students learn how to use proportions to find the lengths of unknown sides in similar polygons. As with finding an unknown length in a pair of similar triangles, a proportion can be set up with a variable as the length of the unknown side. Also as with similar triangles, similar polygons may be overlapping. In this case, students may wish to redraw the similar polygons so they are not overlapping.



2 In a parallelogram, opposite sides are parallel and congruent. Set up proportions that compare the long and short sides of each parallelogram. Notice too, in the larger parallelogram, the length of the longer side is twice the length of the shorter side. The same is true for any parallelogram similar to it. The lengths of the longer sides in the smaller parallelogram are two times four, or eight.

Additional Examples

1. **Find the unknown lengths in the pair of similar figures.**

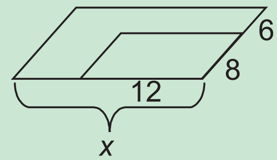


Set up and solve a proportion to find one of the missing side lengths.

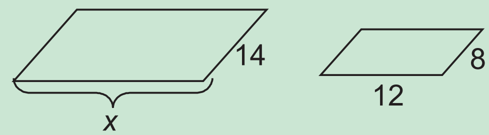
$$\frac{3}{5} = \frac{6}{y}$$
$$3y = 30$$
$$y = 10$$

Since $y = 10$, the triangles are isosceles. Therefore, $x = 6$.

2. **The parallelograms below are similar. Find the value of *x*.**



Separate the parallelograms.



Write and solve a proportion.

$$\frac{x}{12} = \frac{14}{8}$$

$$8x = 168$$
$$x = 21$$

Subtopic 3 Enlargements and Reductions

Expand Their Horizons

In this subtopic, students work with enlargements and reductions. An enlargement or reduction of a figure is the same shape as the original figure, but a different size, so they are similar figures. When making copies on a copier, many people encounter enlargements and reductions that are smaller or larger than the document they are copying.

The ratio of any two corresponding lengths in two similar figures is called the scale factor. If an original four inch by six inch photo is enlarged so that the new sides measure eight inches and 12 inches, the scale factor is 200%, or two, because the ratios of the new sides to the original sides are $\frac{8}{4}$ and $\frac{12}{6}$.

Common Error Alert:

Students may write 50% instead of 200% for a scale factor when the corresponding sides are enlarged from four by six to eight by 12. Tell students to write the sides of the new figure in the numerator and the original figure in the denominator: $\frac{\text{new}}{\text{original}}$. Also point out that a scale factor of 100% is the same as multiplying by one, which results in a congruent figure. Enlargements have a scale factor greater than 100%, and reductions have a scale factor less than 100%.

In the lesson, students solve an enlargement problem two ways: first by multiplying by the scale factor and second by writing a percent proportion. To show how the two methods are related, remind students that the first percent proportion answers the question: *What is 400% of four*? This can be translated into the equation: $x = 400\% \times 4$. To solve, change the percent to a decimal and multiply.

Point out that a scale factor tells how the *side lengths* change. A scale factor of 200% means that the *side lengths* have doubled. It does not mean that the area of the figure has doubled. There is a Challenge Problem that shows how the area is affected.

3 Find 50% of four and then find 50% of six. When writing a percent proportion, the word that follows *of* is the whole. The proportions are $\frac{h}{4} = \frac{50}{100}$ and $\frac{l}{6} = \frac{50}{100}$.

4 Find 700% of eight and 700% of 10. To find 700% of eight, multiply: $7 \times 8 = 56$. To find 700% of 10, multiply: $7 \times 10 = 70$. The enlargement has dimensions of 56 inches by 70 inches.

Additional Examples

1. **Shirley made a 25% copy of a 24 inch by 36 inch photo. What are the dimensions of the reduced figure?**

   Find 25% of 24 and 25% of 36.
   If using percent proportions,

   $$\frac{h}{24} = \frac{25}{100} \text{ and } \frac{h}{36} = \frac{25}{100}$$
   $$\frac{h}{24} = \frac{1}{4} \qquad \frac{h}{36} = \frac{1}{4}$$
   $$\frac{6}{24} = \frac{1\times6}{4\times6} \qquad \frac{9}{36} = \frac{1\times9}{4\times9}$$

   The dimensions are six inches by nine inches.

2. **A 16 in. by 20 in. photo is enlarged. The dimensions of the enlarged photo are 24 in. by 30 in. Write the scale factor as a percent.**

   Find the ratio of the corresponding sides:
   $$\frac{24}{16} = \frac{3}{2}, \ \frac{30}{20} = \frac{3}{2}.$$

   Write the ratio as a percent:
   $$\frac{3}{2} = 1\frac{1}{2} = 1.5 = 150\%.$$

   The scale factor is 150%.

Expand Their Horizons

In this subtopic, students solve problems that involve scale drawings. A scale drawing is a drawing of an object that is similar to the object, but larger or smaller. Because the drawing is similar to the actual object, all corresponding side lengths are proportional. The ratio of a side on the drawing to that side of the actual object is the scale. Because scale drawings are often much smaller or larger than the actual figure, the units in the ratio are often different. Examples are 1 inch to 2 miles or $\frac{1}{4}$ inch = 1 foot.

Students have probably seen scales on maps. Display a page from an atlas on an overhead projector, preferably a page showing cities in or near the school's geographic area, and discuss the scale. Use the scale to estimate the distance between two cities. Then, measure the distance between the cities on the map and then write and solve a proportion to find the actual distance.

Common Error Alert:

Students may confuse scales with scale factors and say that a scale drawing with a scale of 1 inch to 2 feet is a 50% reduction because $\frac{1}{2} = 50\%$. Tell students that to find the scale factor, both units in the ratio must be the same. For a scale of 1 inch to 2 feet, write two feet as 24 inches: $\frac{1 \text{ in.}}{24 \text{ in.}} \approx 4\%$. Teachers may wish to also draw lengths of one inch and two feet on the board to show that the former is far less than 50% of the latter.

5 Write and solve a proportion for each unknown side, using the scale of $\frac{1 \text{ cm}}{2 \text{ m}}$ as one of the ratios: $\frac{1}{2} = \frac{x}{4}$ and $\frac{1}{2} = \frac{y}{6}$. The dimensions on the blueprint are two centimeters by three centimeters.

Additional Examples

1. **A map has a scale of one inch to 200 miles. The distance between two cities on the map is $\frac{3}{4}$ inches. What is the actual distance between the two cities?** Write and solve a proportion.

$$\frac{\text{inches}}{\text{miles}} \rightarrow \frac{1}{200} = \frac{\frac{3}{4}}{x}$$
$$x = 200 \times \frac{3}{4}$$
$$x = 150$$

The actual distance is 150 miles.

2. **A blueprint of a room has a scale of one inch to two feet. The actual door in the room is $2\frac{1}{2}$ feet wide. What is the width of this door on the blueprint?** Write and solve a proportion. (This fraction may be rewritten as a decimal.)

$$\frac{\text{inches}}{\text{feet}} \rightarrow \frac{1}{2} = \frac{x}{2.5}$$
$$2x = 2.5$$
$$x = 1.25$$

The width of the door on the blueprint is 1.25 inches, or $1\frac{1}{4}$ inches, wide.

Look Beyond

Students will study reductions and enlargements again in this course when they study transformations. A transformation is a change. Transformations include slides, rotations, reflections, and dilations. Dilations are reductions and enlargements. As with similar polygons, the ratio of the corresponding sides will be called the scale factor; the original figure will be called the pre-image; and the new figure will be called the image.

Connections

Binoculars are used to make images appear closer. Binoculars are often used at sporting events, concerts, and other live performances as well as for bird-watching. The quality of a pair of binoculars is given by a pair of numbers, in which the first number is the magnification. For example, binoculars labeled 7 × 35 will magnify the image so that it appears to be seven times closer than it actually is. The second number indicates the size of the objective lens, which determines how much light is available for viewing the image. Greater numbers result in brighter images.