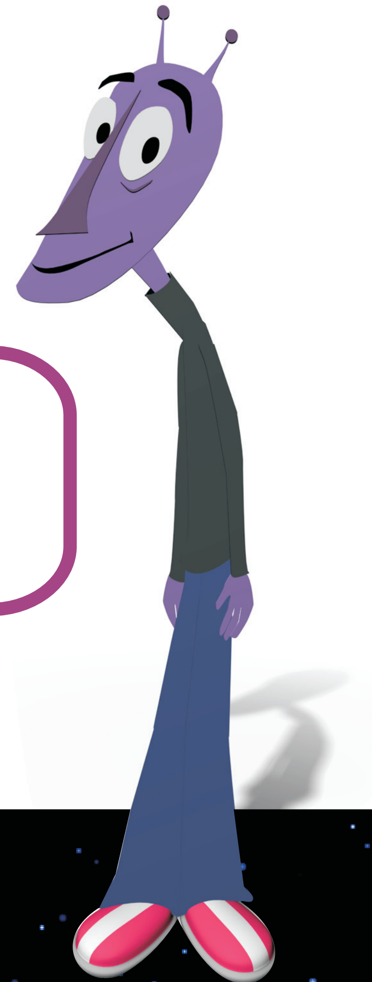
Geometry

★ Module 9 ★

Characteristics of Geometric Shapes



Lesson 3
Circles

Teacher Notes 9.3

Objectives

- Model and identify circle, radius, diameter, center, circumference, and chord.
- Draw, label, and determine relationships among the radius, diameter, center, and circumference (e.g. radius is half the diameter) of a circle.
- Model and develop the concept that *pi* is the ratio of the circumference to the diameter of any circle.

Prerequisites

Identifying and naming line segments

Rounding numbers to a given place value

Multiplying fractions and decimals

Vocabulary

Circle
Equidistant
Plane (8.1)
Center (of a circle)
Radius
Line segment (8.1)
Chord
Diameter
Infinite (3.1)
Congruent (8.2)
Endpoint (8.1)
Circumference
Perimeter (7.1)
Pi
Ratio (4.1)
Terminating decimal (5.5)
Repeating decimal (5.5)
Irrational number
Proportion (7.2)
Commutative Property of Multiplication (1.3)
Improper fraction (4.1)

Get Started

- Divide the class into pairs or groups of three or four. Give each group a compass and a pair of scissors.
- Have each person in the group use the compass to draw a circle. Tell the students to make circles of different sizes and have students use the point of the compass to mark the center of the circle.
- Have each student fold their circle in half and make a crease on the fold. Have them open their circle and then, fold the circle in half again but this time making a different crease line than before. Have students make five or six similar creases. Ask what they notice about these creases. **Possible answers: They all pass through the center mark. They all appear to be the same length.**

- Measure the line segments created by the folds. Ask what they notice. **Possible answers: All the creases are the same length. The center mark divides each crease in half.**

Subtopic 1 Circles

Expand Their Horizons

In this subtopic, students learn how to give the formal definition of a circle, how to name a circle, and how to identify parts of a circle. A circle is defined as a set of points on a plane that are equidistant, or the same distance, from a given point. Relate this definition to the circles the students drew with a compass in the Get Started activity. While drawing a circle, the length of the compass opening did not change. Point out that without the phrase, *on a plane*, the definition would become the definition of a sphere.

Show students that a compass is not the only way to make a circle. Tie a string to a piece of chalk and use it to make a circle on the board by holding the empty end of the string in one place. Show how changing the length of the string changes the size of the circle just as the adjusting the length of a compass opening did. Point out that this distance is the radius. The terms radius and diameter each have two meanings. They can refer to the actual segments or to the lengths of those segments.

A circle is named by its center point. Knowing the center point is important because without it, it would be impossible to identify radii and diameters. In the figure on the left, the segment may appear to be a diameter, but the same figure on the right, which includes the center point, shows that it is not.

Common Error Alert:

Students often confuse the terms *radius* and *diameter*. Tell students that the prefix *dia-* means *through* or *across* and that a diameter goes all the way through, or across, a circle.

For a given circle, the diameter is twice the radius. This translates to the mathematical equation $d = 2r$, which is the formula given in the lesson. Teachers may also want to introduce the formula $r = \frac{1}{2}d$ which translates to *the radius is half the diameter*. Some students may prefer the latter when they know the diameter and are finding the radius.

1 A radius must have one endpoint at *T* and the other endpoint on the circle. $\overline{TR}$ and $\overline{TB}$ are radii. $\overline{RB}$ is a diameter because it connects two points on the circle and passes through *T*. $\overline{RB}$, $\overline{XB}$, and $\overline{XR}$ are chords because they each connect two points on the circle.

2 Point *E* is the center of the circle. Segments $\overline{EB}$, $\overline{EA}$, $\overline{EC}$, and $\overline{ED}$ are radii because they each have *E* as one endpoint and a point on the circle as the other endpoint. $\overline{BC}$ and $\overline{AD}$ are diameters because they each begin and end on the circle and pass through point *E*. Because they are diameters, they are also chords.

3 For a given circle, the radius is half the diameter. Divide 30 by two; the radius is 15 feet.

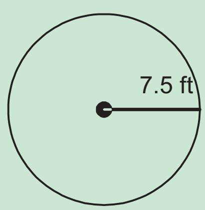
4 A radius can never be a chord because the endpoints of a chord must always lie on the circle while one endpoint of a radius must be the center of the circle. By definition, a diameter is always a chord because its endpoints are on the circle. However, a chord is not always a diameter. A chord is only a diameter if it happens to pass through the center.

Additional Examples

1. A circular swimming pool has a radius of 7.5 feet. What is the diameter of the swimming pool?

The diameter is twice the radius.

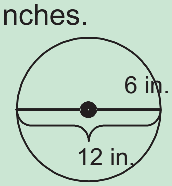
$d = 2r$
$d = 2 \times 7.5$
$d = 15$



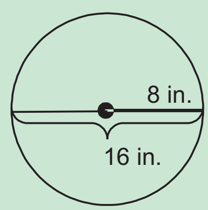
The diameter is 15 feet.

2. Mandy ordered one medium and one large pizza. The diameter of the medium pizza is 12 inches. The radius of the large pizza is two inches greater than the radius of the medium pizza. What is the diameter of the large pizza?

Because the diameter of the medium pizza is 12 inches, the radius is half as much, or six inches.



The radius of the large pizza is two inches greater than the radius of the medium pizza, which makes the radius eight inches and the diameter 16 inches.



The diameter of the large pizza is 16 in.

Expand Their Horizons

In this subtopic, students learn that *pi* is the ratio of the circumference of a circle to its diameter. They also use the formula $C = \pi d$ to find the circumference of a circle.

Pi is an irrational number. Irrational means not rational; therefore, *pi* is a nonrepeating and nonterminating decimal. In other words, it goes on forever without repeating. *Pi* has fascinated mathematicians for over 3,000 years. Many have spent their entire careers approximating the digits of *pi*. The task became easier with the advent of computers. In 2002, *pi* was approximated to 1.2411×10^{12} digits, which is greater than one trillion digits.

In this lesson, *pi* is approximated as 3.14 or $\frac{22}{7}$. Teachers may wish to show students the π button on a calculator. Because of the screen space, only the first 10 to 12 digits can be seen. Any calculation that uses *pi* is an approximation.

Diameter of circle: 5 ft.
Exact circumference of circle: 5π ft.
Approximate circumference of circle: 5(3.14) = 15.7 ft.

As more digits of *pi* are used, the closer the approximation becomes to the actual answer. Circumference, like perimeter, is measured in linear units.

If given the option of which approximation to use, students may wish to use the fraction when the diameter is a multiple of seven. This way, the fractions can be simplified.

In the lesson, the formula $C = \pi d$ is given. When students know the radius of a circle, they must double it to find the diameter before using the formula. An option is to rewrite the formula with $2r$ in place of d: $C = \pi\,(2r)$ or $C = 2\pi\, r$.

5 Use the formula $C = \pi d$. The diameter of the wheel is 28 inches, so $C = \pi \times 28$. Use 3.14 to approximate π. $C = 3.14 \times 28 \approx 87.92$ inches. Rounding the answer to the nearest whole number gives a circumference of approximately 88 inches.

Because 28 is a multiple of seven, students may choose to use $\frac{22}{7}$ for π.

$$C = \frac{22}{\cancel{7}_1} \times \cancel{28}^4 \approx 88$$

6 Because the diameter is expressed as a fraction, $\frac{22}{7}$ is an appropriate approximation for π. Change $2\frac{1}{2}$ to the improper fraction $\frac{5}{2}$ and multiply it by $\frac{22}{7}$. The circumference is approximately $7\frac{6}{7}$ feet. To use 3.14 for π, write $2\frac{1}{2}$ as 2.5.

Additional Examples

1. A DVD has a radius of $2\frac{5}{16}$ inches. What is the circumference of the DVD?

Double the radius to find the diameter. Then, use the formula $C = \pi d$.

$$d = 2 \times 2\frac{5}{16}$$
$$= 2 \times \frac{37}{16}$$
$$= \frac{37}{8}$$

$$C = \pi d$$
$$\approx \frac{22}{7} \times \frac{37}{8}$$
$$\approx \frac{407}{28}$$
$$\approx 14\frac{15}{28}$$

The circumference is about $14\frac{15}{28}$ inches.

2. The circumference of a Ferris wheel is approximately 276 feet. What is the diameter of the Ferris wheel?

Use the formula $C = \pi d$. Substitute 276 for C and 3.14 for π. Then, solve for d.

$$C = \pi d$$
$$276 \approx 3.14 \times d$$
$$\frac{276}{3.14} \approx d$$
$$88 \approx d$$

The diameter is approximately 88 feet.

Look Beyond

Students will use *pi* later in this course when they find the area of a circle. The area of a circle is *pi* times the square of the radius. They will also use *pi* to find the surface area and volume of cones, cylinders, and spheres.

In high school geometry, students will study circles and parts of circles in greater detail. They will explore the angles formed by radii and chords, and they will learn about secants and tangents, which are lines that intersect a circle. In statistics, students will use circle graphs to display data. Each category will be a sector of a circle.

Connections

Tires for cars and light trucks come in different sizes because the wheels are different sizes. Most tires have a code on the side, where the last number given is the diameter of the wheel rim in inches. This number often follows the letter R, but this R does not stand for radius. It means the tire is a radial tire. A common tire code looks like this: P225-75R14. A tire with this code fits a wheel with a diameter of 14 inches.