Geometry

Module 9

Characteristics of Geometric Shapes

Lesson 2
Quadrilaterals

Teacher Notes 9.2

Objectives

- Classify quadrilaterals.
- Use paper and physical models to determine the sum of the measures of interior angles of quadrilaterals.
- Find the missing measure of a quadrilateral.
- Compare quadrilaterals.

Prerequisites

Identifying simple polygons, including quadrilaterals

Identifying parallel and perpendicular line segments

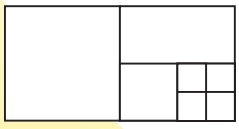
Writing and solving equations with one variable

Vocabulary

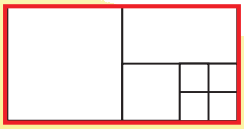
Quadrilateral (9.1)
Polygon (9.1)
Square
Parallelogram
Parallel lines (8.2)
Rectangle
Right angle (8.2)
Rhombus
Congruent (8.2)
Trapezoid
Triangle Sum Property (8.4)
Triangle (8.4)
Quadrilateral Sum Property

Get Started

- Draw the following diagram on the board:



- Ask students how many quadrilaterals are in the diagram. If they answer "seven," show them a quadrilateral that encloses other quadrilaterals, such as the largest one.



- There are 15 quadrilaterals. Show students how to find each quadrilateral using a systematic process. For example, there is one large rectangle, as shown in diagram. Two are half that size; two are half of that size; two are half of that size; four are half of that size (two vertically split and two horizontally split); and then there are the four smallest quadrilaterals.

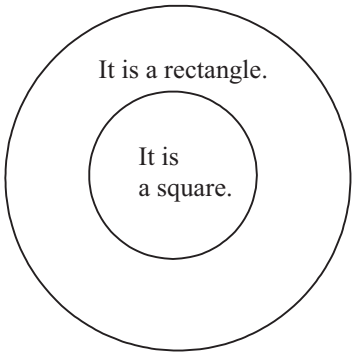
Subtopic 1 Types of Quadrilaterals

Expand Their Horizons

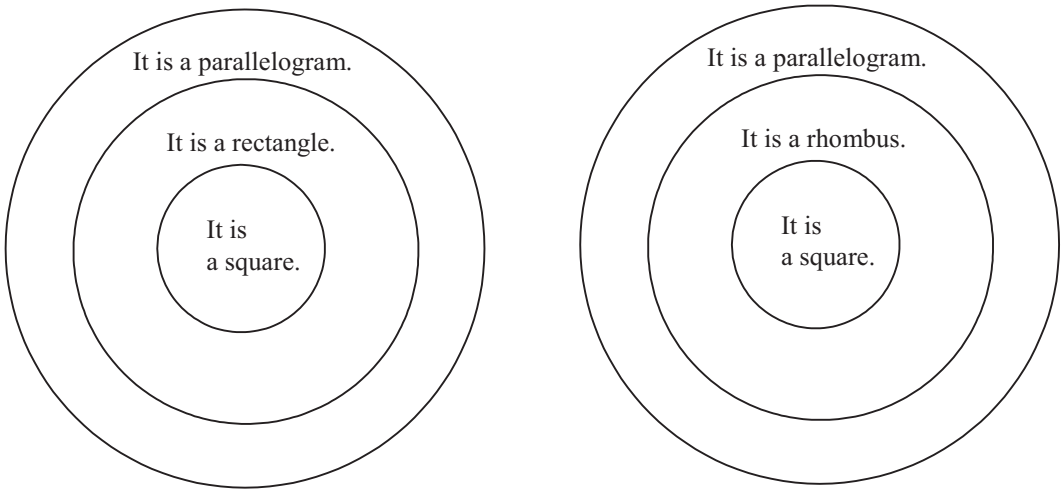
In this subtopic, students are reminded that a quadrilateral is a four-sided polygon. As with any polygon, it can be named by listing all of the vertices in consecutive order. In other words, begin with any vertex and then, list the remaining vertices by going around the figure in either direction. Point out that this rule actually applies to triangles as well, but because there are only three vertices, listing the three vertices in any order accomplishes the same result.

If both pairs of opposite sides are parallel to each other, then the quadrilateral is a parallelogram. There are different types of parallelograms. A parallelogram with four right angles is a rectangle. A parallelogram with four congruent sides is a rhombus. A parallelogram with both four right angles and four congruent sides is a square. Point out that this makes a square a regular polygon.

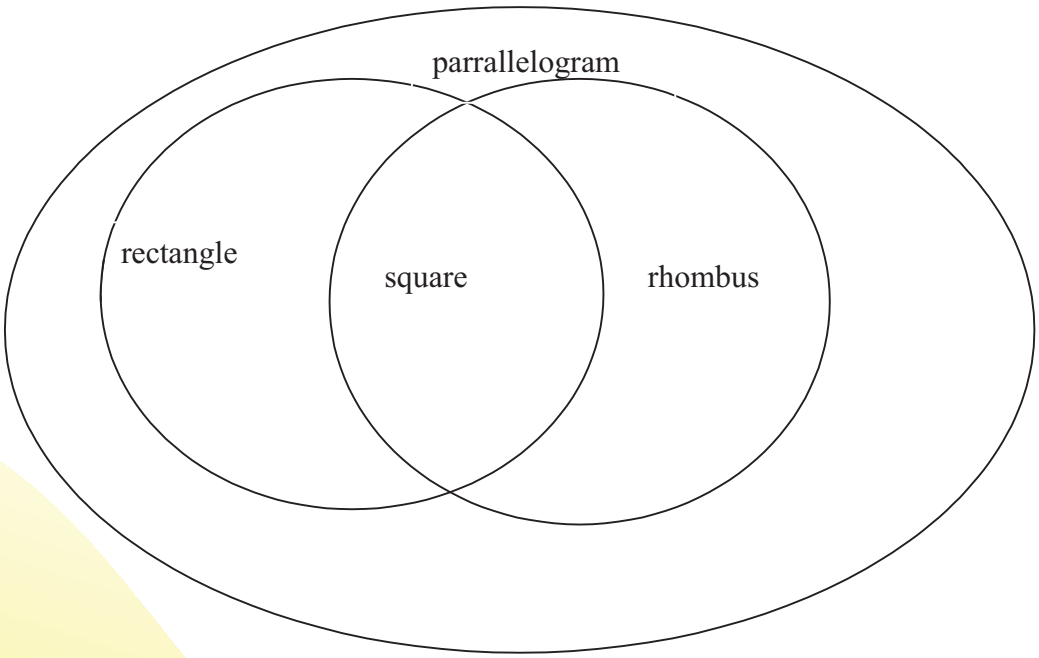
A Euler diagram can be used to help sort out this information. In the Euler diagram below, any point in the interior of the inner circle is also in the interior of the outer circle. So it is true to say, *If it is a square, then it is a rectangle*. On the other hand, a point in the interior of the outer circle may or may not be in the interior of the inner circle, so it is not necessarily true that, *If it is a rectangle, then it is a square*.



The following two Euler diagrams summarize the relationships among parallelograms:



Shown below is the Venn diagram which may also be used to summarize the relationship among parallelograms.



If only one pair of opposite sides is parallel, then the quadrilateral is a trapezoid. A trapezoid is never a parallelogram, and a parallelogram is never a trapezoid.

1 Start with the broadest classification and work down. It is a quadrilateral because it is a four-sided polygon. It is a parallelogram because opposite sides are parallel, and it is a rectangle because all the angles are right angles. To name the figure, start with any vertex and list the other three vertices in order. Examples include *MNTR*, *TNMR*, *RMNT*, *NTRM* and *MRTN*.

2 The figure is a four-sided polygon. It is a quadrilateral. No sides are parallel, so it cannot be classified any other way.

3 The first figure has two pairs of parallel sides, but the angles are not right angles, and the sides are not congruent. It is a parallelogram. The second figure has exactly one pair of parallel sides. It is a trapezoid. The third figure is a parallelogram with four right angles which makes it a rectangle.

4 By definition, every rectangle is a parallelogram. The first statement is true. A parallelogram is not a trapezoid, and a square is a parallelogram. The second statement is true. A quadrilateral is any simple four-sided polygon. It does not require that any sides be parallel. The last statement is false.

Additional Examples

1. If FRED is an acceptable name for a rectangle, then which side is parallel to $\overline{RE}$?

Draw a rectangle and label the vertices in consecutive order.

$\overline{FD}$ is parallel to $\overline{RE}$.

2. Write always, sometimes, or never.

- **A square is a rhombus.**
- **A rhombus is a square.**

The first statement is *always* true. A rhombus is a parallelogram with four congruent sides, and a square is a parallelogram with four congruent sides and four right angles.

The second statement is *sometimes* true. A rhombus does not need to have four right angles, but if it does, it is also a square.

Subtopic 2 Angles of Quadrilaterals

Expand Their Horizons

In this subtopic, students learn the Quadrilateral Sum Property which states that the interior measures of any quadrilateral have a sum of 360°. This is easily shown by dividing a quadrilateral into two triangles. The sum of the interior measures of any triangle is 180°. It follows that the sum of the interior measures of two triangles will be twice that, or 360°.

If students do not wish to write an equation, they can add the values of the three given angles and subtract that sum from 360°.

5 The three angles with known measures have an angle sum of 220°. Subtract this amount from 360°. The result is the angle measure of the fourth angle, 140°.

Students may notice that the opposite sides of any parallelogram are always congruent.

6 Recall that a right angle measures 90°. This figure has two right angles. (adding 180° to the third angle measure gives 295° and subtracting from 360° gives 65° for the fourth angle measure)

Common Error Alert:

When an angle is marked as either congruent or right, rather than with a numerical angle measure, students may not use that angle measure when finding the sum of the known angle measures. Suggest that students write the following, and then fill in the missing values:

_____ + _____ + _____ + _____ = 360°.

This will help ensure they have taken all four angles into account.

Additional Examples

1. Three angles of a quadrilateral measure 50°, 95°, and 140°. What is the measure of the fourth angle?

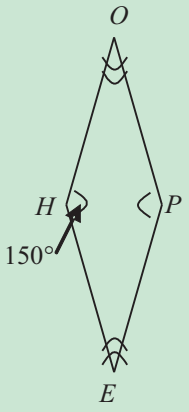
Add the three known measures:
50° + 95° + 140° = 285°.

Subtract from 360° to find the missing measure:
360° – 285° = 75°.

The fourth angle measures 75°.

2. In rhombus *HOPE*, $\angle H \cong \angle P$, $\angle O \cong \angle E$, and $m\angle H = 150°$. Find $m\angle E$.

Draw and label the figure.



$m\angle P = 150°$
$m\angle H + m\angle P = 300°$
Subtract from 360°: 360° – 300° = 60°.
Divide by two because the two remaining angles are congruent: 60° ÷ 2 = 30°.
$m\angle E = 30°$.

Look Beyond

Students have now studied polygons, including an in-depth look at triangles and quadrilaterals. Later, students will study three-dimensional figures. Polyhedra are three-dimensional figures whose sides are polygons. A cone is not a polyhedron because one of its sides is a circle, and a circle is not a polygon.

Polyhedra include prisms and pyramids. Prisms have a pair of parallel sides called bases. Pyramids have only one base, which can be any polygon, and whose other sides are triangles.

Connections

Most foundations for buildings are in the shape of a rectangle. To outline the location of a building, a construction worker can place stakes at each corner and tie them together with string. One property that a rectangle has, that a parallelogram does not have, is that its diagonals are congruent. The diagonals are the segments that connect opposite vertices. The construction worker can measure the distance of the diagonals to check that the figure outline is indeed rectangular and that the corners are square (making 90° angles).