## Geometry

## $\star$ Module 9 *

## Characteristics of Geometric Shapes

## Lesson 1 <br> Polygons

## Objectives

- Identify and model regular and irregular polygons including decagon.
- Identify and model convex and concave polygons.
- Identify, draw, classify, and compare geometric figures using models and real-world examples.


## Prerequisites

## Get Started

- Draw two closed figures and two open figures on the board and have students make up a definition for a closed figure. Accept any reasonable answer.

Closed


Open
$7 \bowtie$

- Draw the following figure on the board:

- Have students use colored chalk to trace both closed and open figures with three, four, five, six, and seven sides. Possible answers for closed five-, six-, and seven-sided figures and an open seven-sided figure are shown below:



## Subtapic 1 <br> Convex and Concave Polygons

## Expand Their Horizons

In this subtopic, students learn the definition of a polygon; they learn the difference between simple and complex polygons; and they learn how to distinguish between a convex and concave polygon. A polygon is a closed plane figure made up of line segments that intersect at their endpoints. By being closed, the polygon has a distinct interior and exterior. It lies entirely in one plane, which makes it two-dimensional. A twodimensional figure has length and width, but no height. Each side is a line segment; no side is a curve. If the sides do not cross each other, the polygon is a simple polygon. Most of the polygons the students will study in this course will be simple polygons.

Polygons can be classified as either convex or concave. If a polygon is convex, then any two points in the interior of the polygon can be connected by a line segment that is also completely in the interior of the polygon.


If a polygon is concave, there exist two points in the interior of the polygon such that the line segment that connects them is not entirely in the interior of the polygon.



A concave polygon can also be defined as a polygon with a side that contains a line that passes through the interior of the polygon. For the concave examples above, notice that each contains at least one side, that when extended, passes through the interior of the polygon. This is not true for any of the sides of a convex polygon.


The sides are not line segments because they are curved. It is not a polygon.

A line segment lies partly outside the polygon. The figure is concave.


Additional Examples

1. Tell if the polygon is simple or complex.


The sides do not cross each other. It is a simple polygon.
2. Tell if the polygon is convex or concave.


No side, when extended, would pass through the interior of the figure.


The polygon is convex.

## Subtrpic 己

## Classifying Polygons According to Sides

## Expand Their Horizons

In this subtopic, students classify polygons according to the number of sides. A polygon can have no fewer than three sides because two distinct segments cannot be joined to form a closed figure. There is no upper limit to the number of sides a polygon can have. Show polygons of increasing numbers of sides in a row and ask students if they notice anything about the shape of the figure as the number of sides increase.


As the number of sides increase, the polygon looks more and more like a circle. It never becomes a circle, although with many sides, it is hard to tell the difference because the length of each side is extremely small. Note that this only works when the polygons are convex and the sides and angles are congruent.

Students learn names for polygons up to 10 sides. Tell students that a polygon with any number of sides can be named by giving the number of sides followed by -gon. For example, a polygon with 15 sides is a 15 -gon.

Count the number of sides. It is five. A five-sided polygon is a pentagon.

Identify the figures with six sides. They are $C$ and $D$. Figure $D$ is convex. The answer is $C$.

## Additional Examples

1. Classify the figure by its number of sides.


The figure has 10 sides. It is a decagon.

## 2. Draw a concave heptagon.

Draw a figure with seven sides so that at least one side, when extended, passes through the interior. Possible answer:


## Subtapic ヨ <br> Regular and Irregular Polygons

## Expand Their Horizons

In this subtopic, students learn the difference between regular and irregular polygons. In a regular polygon, all of the sides are congruent, and all of the angles are congruent.

## Common Error Alert:

Students may think that a polygon with only congruent sides or with only congruent angles is enough information to say that the polygon is regular. Point out that both the sides and angles must be congruent, because one condition can be true when the other is not. In a rectangle, all the angles are congruent, but not all the sides are congruent.


In this quadrilateral, which students will later learn is called a rhombus, all the sides are congruent, but all the angles are not congruent.


A square has four equal sides and four right angles. A square is a regular polygon.

## Additional Examples

1. Tell if the polygon is regular or irregular.


Two sides have a different length than the other two sides. Since all the sides are not the same length, the polygon is irregular.
2. A regular pentagon has a perimeter of 75 centimeters. What is the length of each side?

A pentagon has five sides. Since it is regular, all of the five sides are congruent. Divide to find the length of each side: $75 \div 5=15$.

Each side is 15 centimeters long.

## Look Beyond

In the next lesson, students will further investigate the properties of quadrilaterals. They will classify quadrilaterals based on how many pairs of sides are parallel. They will also classify quadrilaterals based on whether the sides and angles are congruent. They will also determine the sum of the interior angle measures of any quadrilateral. Later in the course, they will learn how to find the areas of these figures.

In more advanced classes, students will find both interior and exterior angle measures for polygons with more than four sides. They will also learn methods for finding the areas of these polygons.

## Connections

The prefixes in the names of the commonly used polygons are prefixes that are used in everyday life. For example, quadruplets are a set of four children; an octopus has eight tentacles; and a decathlon has ten events.

Chemists study the structure of molecules. Atoms bond to each other in two- and threedimensional formations. The names of the compounds formed are often based on the angle measures formed by the bonds and the shape of the figure, which may be polygonal, in whole or in part. Examples include cyclohexane, cycloheptane, and neopentyl bromide.

