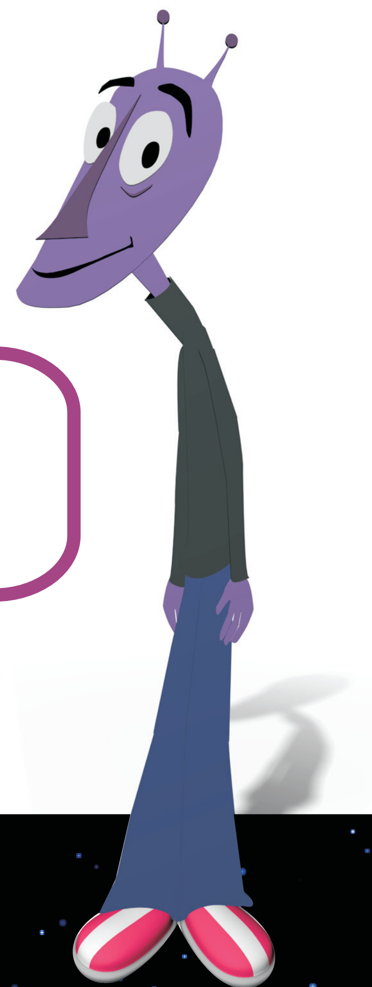
Geometry

Module 8

Points, Lines, Angles, and Triangles



Lesson 7
Right Triangles

Teacher Notes 8.7

Objectives

- Prove and use the Pythagorean Theorem and its converse.
- Use special right triangles to solve real-life problems.

Prerequisites

Squaring numbers

Finding the square roots of perfect square numbers

Rounding decimals

Vocabulary

Pythagorean Theorem
Triangle (8.4)
Right triangle (8.4)
Right angle (8.2)
Hypotenuse
Leg
Square number (7.1)
Area (7.1)
Perfect square number (7.1)
Square root (7.1)
Converse

Get Started

- Have students evaluate $\sqrt{16}$ and $\sqrt{25}$. **$\sqrt{16} = 4$, $\sqrt{25} = 5$**
- Have students estimate $\sqrt{20}$. Have them explain their reasoning.
  The actual answer is an irrational number. Students may say the square root is about 4.5 because $\sqrt{20}$ is about half way between $\sqrt{16}$ and $\sqrt{25}$.

Discuss how $\sqrt{24}$ would be closer to five and how $\sqrt{17}$ would be closer to four.

- Divide the students into two teams. Write a square root of a number that is not a perfect square on the board and have each team confer quietly and decide on an estimate, to the nearest tenth. No calculators are allowed. The team with the closer estimate to the actual answer gets one point.

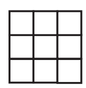
- Play until one team gets a certain number of points, perhaps 10.

Subtopic 1 — The Pythagorean Theorem

Expand Their Horizons

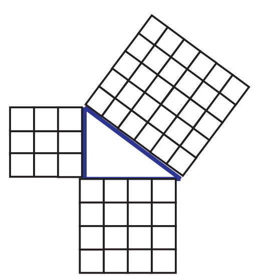
In this subtopic, the parts of a right triangle are reviewed, and the Pythagorean Theorem is introduced. Although there is evidence that the theorem was used before the time of Pythagoras, the theorem is still named after him because he is believed to be the first person to formally prove it. There are several ways to prove the Pythagorean Theorem. Students may be interested to know that former president James Garfield wrote his own proof of the Pythagorean Theorem.

Remind students that a square number can be modeled by a square. For instance, nine is a square number because it can be modeled by three rows and three columns.

9 

The square root of a perfect square number is the length of the side of its square. The square root of nine is three because each side in the diagram above has a length of three units.

Students are shown the following diagram of a right triangle, where the lengths of the sides of a triangle are three, four, and five.



The side of length three is the side of a square with an area of nine and $3^2 = 9$. The side of length four is the side of a square with an area of 16, and $4^2 = 16$. The side of length five is the side of a square with an area of 25, and $5^2 = 25$. Show that the area of the two smaller squares equals the area of the big square: $9 + 16 = 25$. Compare this “layman’s statement” to the statement: In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. This is true for all right triangles. Teachers may wish to have students use graph paper to make a similar diagram for a right triangle of side lengths five, 12, and 13.

The Pythagorean Theorem is also shown as $a^2 + b^2 = c^2$ where *a* and *b* are the lengths of the legs and *c* is the length of the hypotenuse. It does not matter which leg is *a* and which leg is *b*. For that reason, the formula is sometimes written as $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$.

Common Error Alert:

Students often write the missing side as *c*, even if the missing side is a leg. Stress to the students that the missing side could be either a leg or the hypotenuse. It may help students to have their very first step in a problem to be to identify the hypotenuse by locating the right angle and by drawing a line to the opposite side and by labeling this side as *c*.

When solving for an unknown, students will need to find square roots. For example, if $b^2 = 64$, then *b* is the square root of 64, which is eight, because $8^2 = 64$. Often times, however, the number will not be a perfect square number. Have each student find the square root button on their calculator. It will look something like $\boxed{\sqrt{x}}$. Some calculators may require that they first press a Shift or Second key. The square root key is often near a key for squaring a number which would look something like $\boxed{x^2}$. Before solving problems, have students practice squaring numbers and finding square roots on their calculators.

1 Sketch a diagram of a house and a ladder. Assume the house is perpendicular to the ground. The *base* of the ladder means the *bottom* of the ladder. This forms a right triangle where the ladder is the hypotenuse because it is the side opposite the right angle. In $a^2 + b^2 = c^2$, substitute 12 for either *a* or *b* and substitute 20 for *c*. This gives either $12^2 + b^2 = 20^2$ or $a^2 + 12^2 = 20^2$. Evaluate the squares and the equation becomes $144 + b^2 = 400$ or $a^2 + 144 = 400$. To find the unknown, subtract 144 from 400. The equation is now either $b^2 = 256$ or $a^2 = 256$. Notice that in both cases the result is the same, students must find $\sqrt{256}$, which is 16 feet.

2 Sketch a diagram. The distances Martha walked make up the legs of a right triangle. The direct distance between the station and camp is the hypotenuse. Substitute eight and 12 for *a* and *b*: $8^2 + 12^2 = c^2$ or $12^2 + 8^2 = c^2$. Square eight and 12 and add the squares together. Both equations give the same result: $208 = c^2$. Use a calculator to find $\sqrt{208}$ and round to the nearest hundredth: about 14.42 mi.

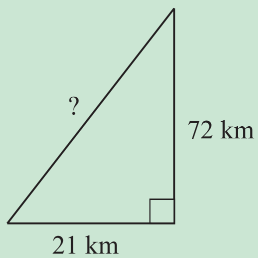
Ask students if they ever heard the saying, “The shortest distance between two points is a straight line.” Point out how this applies to this situation and in the case of Zeo’s shortcut.

3 Sketch a diagram. The base of the pole is the bottom of the pole. The missing side is a leg. Set up the equation with eight for *a* or *b* and 30 for *c*. Square each number and subtract 64 from 900. Find $\sqrt{836}$ and round to the nearest whole number or foot.

Additional Examples

1. **From home, Jaime drove 21 kilometers due east and 72 kilometers due north. How far from home is Jaime now?**

   Sketch a diagram.



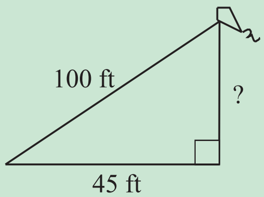
The missing distance is the hypotenuse of a right triangle.

$$a^2 + b^2 = c^2$$
$$21^2 + 72^2 = c^2$$
$$441 + 5{,}184 = c^2$$
$$5{,}625 = c^2$$
$$75 = c$$

Jaime is 75 kilometers from home.

2. **A kite is flying at the end of 100 feet of string and secured to the ground with a stake. The distance between the stake and the spot on the ground directly below the kite is 45 feet. To the nearest foot, how high above the ground is the kite?**

   Sketch a diagram.



The missing distance is the leg of a right triangle.

$$a^2 + b^2 = c^2$$
$$45^2 + b^2 = 100^2$$
$$2{,}025 + b^2 = 10{,}000$$
$$b^2 = 7{,}975$$
$$b \approx 89$$

The kite is about 89 feet above the ground.

Subtopic 2 — Using the Converse of the Pythagorean Theorem

Expand Their Horizons

In this subtopic, students learn the converse of the Pythagorean Theorem. A converse is a statement that switches the "if" phrase with the "then" phrase. For example, the converse of *If it is sunny, then I will swim*, is *If I swim, then it is sunny*. The converse of an if-then statement is not always true. The converse of the Pythagorean Theorem though, is always true.

Pythagorean Theorem: If a triangle is a right triangle, then $a^2 + b^2 = c^2$.
Converse of Pythagorean Theorem: If $a^2 + b^2 = c^2$, then the triangle is a right triangle.

To check that a triangle is a right triangle, substitute the side lengths into $a^2 + b^2 = c^2$. Because the hypotenuse is the longest side of a triangle, substitute the longest length for c. Substitute the other two lengths for a and b.

If $a^2 + b^2$ do not equal c^2, then the triangle is not a right triangle. It is either acute or obtuse. Teachers may wish to point out that if c^2 is greater than $a^2 + b^2$, the triangle is obtuse. If c^2 is less than $a^2 + b^2$, the triangle is acute.

4 Because 25 is the longest length, substitute 25 for c. Substitute seven and 24 for a and b. Evaluate each side: $a^2 + b^2 = 625$ and $c^2 = 625$. The triangle is a right triangle because $a^2 + b^2 = c^2$.

Additional Examples

1. The lengths of the sides of a triangle are 18, 24, and 30 inches. Is this a right triangle?

Substitute 30 for c and 18 and 24 for a and b. Then, evaluate both sides of the equation.

$$a^2 + b^2 \stackrel{?}{=} c^2$$

$$18^2 + 24^2 \stackrel{?}{=} 30^2$$

$$324 + 576 \stackrel{?}{=} 900$$
$$900 = 900$$

Yes

2. The lengths of the sides of a triangle are six, 10, and 12 inches. Is this a right triangle?

Substitute 12 for c and six and 10 for a and b. Then, evaluate both sides of the equation.

$$a^2 + b^2 \stackrel{?}{=} c^2$$

$$6^2 + 10^2 \stackrel{?}{=} 12^2$$

$$36 + 100 \stackrel{?}{=} 144$$
$$136 \neq 144$$

No

Look Beyond

The Pythagorean Theorem can be used to prove the relationships that exist in the special right triangles, which are the 45°-45°-90° triangle and the 30°-60°-90° triangle.

A 45°-45°-90° triangle is a triangle with angle measures of 45°, 45°, and 90°. Because two angles are congruent, two sides are congruent, so every 45°-45°-90° triangle is an isosceles right triangle. In an isosceles right triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg. This can be shown by applying the Pythagorean Theorem to an isosceles right triangle whose side lengths are one:

$$1^2 + 1^2 = c^2$$
$$1 + 1 = c^2$$
$$2 = c^2$$
$$\sqrt{2} = c.$$

Connections

Construction workers check that corners are at right angles by applying the “3-4-5” rule. From the corner, they measure three inches along one side and place a mark. Then they measure from the corner along the other side four inches and place a mark. Last, they measure the distance between the two marks. If it is not five inches, the angle is not a right angle, and the corner is not square. It is especially important that the corners of foundations of buildings be square so that the walls can be built to meet at their edges and that the ceilings lie parallel to the floor.