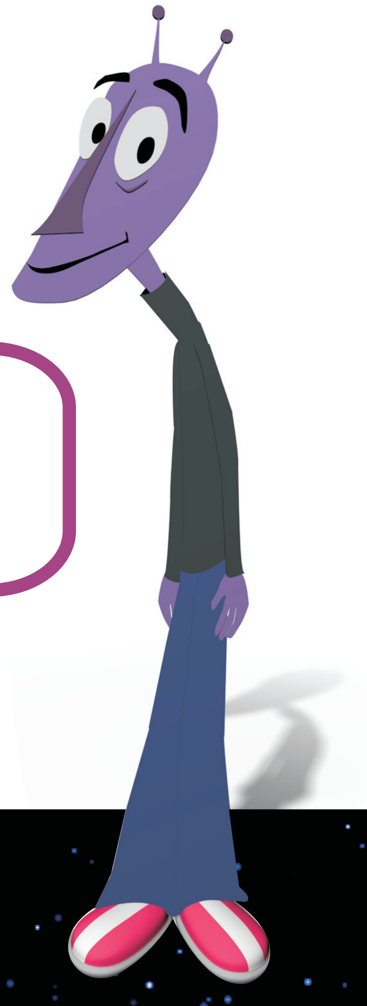
Geometry

★ Module 8 ★

Points, Lines, Angles, and Triangles



Lesson 6
Similar Triangles

Teacher Notes 8.6

Objectives

- Determine if triangles are similar.
- Develop the properties of similar triangles (ratio of sides and congruent angles).
- Use similar triangles to solve problems.

Prerequisites

Identifying corresponding parts of triangles

Using the Triangle Sum Property

Writing and simplifying ratios

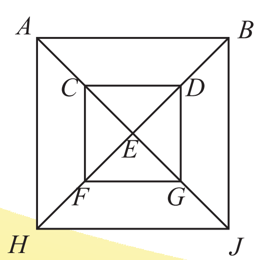
Writing and solving proportions

Vocabulary

Similar triangles
Triangle (8.4)
Right angle (8.2)
Congruent (8.2)
Acute angle (8.2)
Corresponding parts (8.5)
Angle (8.1)
Proportion (7.2)
Ratio (4.1)
Similarity statement
Angle-Angle Similarity
Side-Side-Side Similarity
Triangle Sum Property (8.4)
Vertex of a triangle (8.4)
Indirect measurement

Get Started

- Draw the following diagram on the board.



- Have students name all the triangles that appear congruent to $\triangle AEB$.
  $\triangle BEJ$, $\triangle JEH$ and $\triangle HEA$

- Have students name all the triangles that appear congruent to $\triangle CED$.
  $\triangle DEG$, $\triangle GEF$ and $\triangle FEC$

- Ask students how the first set of triangles compares to the second set.
  Possible answer: The second set have the same shape as the first, but they are smaller.

- Have students find other triangles that also appear to be the same shape but different size. **Possible answer: $\triangle AHJ$ and $\triangle CFG$**

- Tell students today's lesson will be about properties that are shared by triangles that are the same shape but different sizes.

Subtopic 1 — Similar Triangles

Expand Their Horizons

In this subtopic, students learn the definition and properties of similar triangles and how to prove that two triangles are similar. Similar triangles are triangles that have the same shape. They may or may not be the same size, which means that congruent triangles are a subset of similar triangles.

In similar triangles, each pair of corresponding angles is congruent, and all the corresponding side lengths are proportional. This means that the ratio of each pair of corresponding sides simplifies to the same value. This value is called the scale factor. Students will learn this term and will use it to solve problems in the next module when they learn about similar polygons.

Just as a congruence statement is written for congruent triangles, a similarity statement is written for similar triangles. Instead of using the congruent symbol ($\cong$) between the triangles, the symbol for *is similar to* ($\sim$) is used. As with congruence statements, it is of utmost importance that the vertices be written in corresponding order.

To prove that two triangles are similar, either show that two pairs of angles are congruent (Angle-Angle, or AA Similarity), or show that all three corresponding side lengths are proportional (Side-Side-Side, or SSS Similarity). To use SSS Similarity, find the ratio of all three corresponding sides. If the ratios are the same (when simplified), the triangles are similar.

Common Error Alert:

Students may not write the word *Similarity* after writing SSS. Remind them that without the word *Similarity*, the reason may be confused with SSS *Congruence*, which states that the sides are congruent, not proportional.

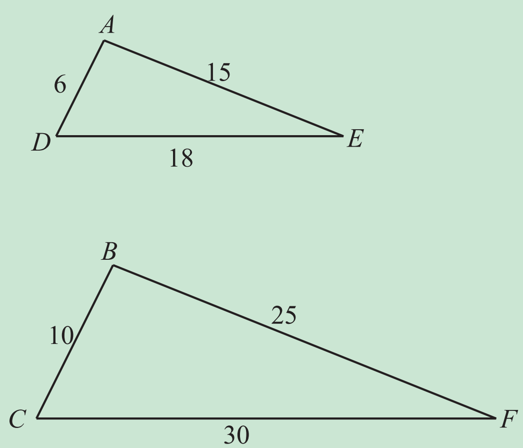
1 Look at the similarity statement to see which angles are corresponding angles. Teachers may wish to redraw the second figure rotated to be in the same position as the first. Use either the new drawing or the similarity statement to find the corresponding sides. Like congruence statements, the endpoints are listed in the same relative position in the similarity statement. The corresponding sides are proportional.

2 The angles are congruent, so the triangles are similar by AA Similarity. If checking the sides, each ratio of corresponding sides reduces to $\frac{3}{2}$. This makes the triangles similar by SSS Similarity.

If choosing to introduce scale factor now, the scale factor between the triangles is $\frac{3}{2}$, or 150%. This means each side of the larger triangle is 150% the length of the corresponding side in the smaller triangle. Students may also choose to write each ratio with the lengths of the smaller triangle in the numerator and the lengths of the larger triangle in the denominator. In this case, each ratio reduces to $\frac{2}{3}$. Each side of the smaller triangle is two-thirds the length of the corresponding side in the larger triangle.

Additional Examples

1. Determine if the triangles are similar. If so, write the similarity statement.

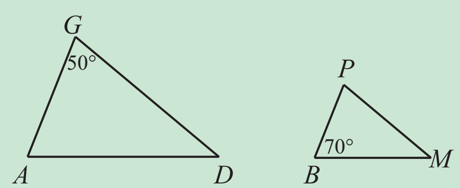


The angle measures are not given and cannot be determined. Form a ratio from each pair of corresponding sides.

$$\frac{AD}{BC} = \frac{6}{10} = \frac{3}{5}, \quad \frac{AE}{BF} = \frac{15}{25} = \frac{3}{5}, \quad \frac{DE}{CF} = \frac{18}{30} = \frac{3}{5}$$

The ratios are congruent. The triangles are similar. To write the similarity statement, list the vertices of each triangle in corresponding order: $\triangle ADE \sim \triangle BCF$.

2. $\triangle GAD \sim \triangle PBM$. Find $m\angle D$.



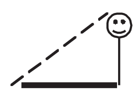
In similar triangles, the corresponding angles are congruent. Because $\angle A$ corresponds to $\angle B$, $m\angle A = 70°$. By the Triangle Sum Property, $m\angle D = 180° - (70° + 50°)$, so $m\angle D = 60°$.

Expand Their Horizons

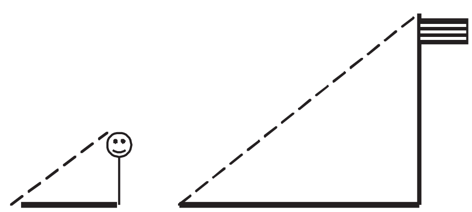
In this subtopic, students use the properties of similar triangles to find missing measures which represent missing distances. Because the distances are not being found directly (such as by aligning a measuring tool like a ruler along side of it), the distances are said to be measured indirectly. Indirect measurement is helpful when the distances would be difficult, dangerous, or impossible to measure directly, such as finding the distance across a lake or finding the height of a tall object.

To use similar triangles to make indirect measurements, distances which are easy to find, such as those along dry land, must be measured. The distances must also form similar triangles, where one pair of known distances form corresponding sides and the corresponding part of the unknown distance is given. Then a proportion is written and solved.

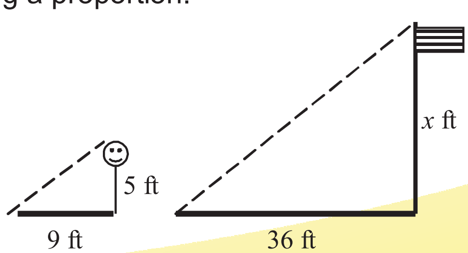
One of the most common scenarios for using similar triangles is to use a person's shadow and a tall object's shadow to find the height of the tall object. A triangle can be formed using the person, the person's shadow, and the length from the end of the shadow to the person's head.



Another triangle is then formed using the tall object (we will use a flagpole), the flagpole's shadow, and the length from the end of the shadow to the top of the flagpole.



When the shadows are measured at the same time of day, the acute angles of the right triangles are congruent. The right angles are congruent because they both measure 90°. The triangles are similar by AA Similarity. The shadows can easily be measured with a tape measure because they are on the ground. The person's height is either known or can easily be measured as well. The height of the flagpole can be found by writing and by solving a proportion.



$$\frac{shadow}{height}: \frac{9}{5} = \frac{36}{x}$$

$$\frac{9 \times 4}{5 \times 4} = \frac{36}{20}$$

The height of the flagpole is 20 feet. Notice that the following proportions are also correct: $\frac{9}{36} = \frac{5}{x}$ and $\frac{5}{9} = \frac{x}{36}$.

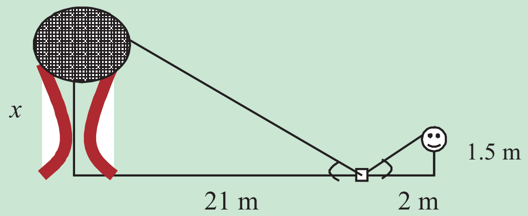
If possible, take students outside on a sunny day and determine the height of a tall object, such as the school flagpole or a tall tree. Encourage students to try this method at home during different times of the day when shadows are of different lengths.

3 The triangles are similar by AA Similarity. Possible proportions are $\frac{3}{7} = \frac{6}{h}$ and $\frac{3}{6} = \frac{7}{h}$. Solve the proportion to find that h = 14 feet.

4 The triangles are similar by AA Similarity. To be sure the correct side lengths are chosen, it may help to redraw the two triangles separately. Possible proportions are $\frac{3}{18} = \frac{4}{h}$ and $\frac{3}{4} = \frac{18}{h}$. Solve the proportion to find that h = 24 feet.

Additional Examples

1. **Ronen calculated the height of a tree by placing a mirror on the ground and by positioning himself at a distance that allowed him to see the top of the tree in the mirror.**



Use the diagram to find the height of the tree.

2. **An eight-foot high fence casts a six-foot long shadow at the same time of day a nearby building casts a 24-foot long shadow. Draw similar triangles to illustrate the situation. Then, find the height of the building.**

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The triangles are similar by AA Similarity. Write and solve a proportion using ratios of corresponding sides.

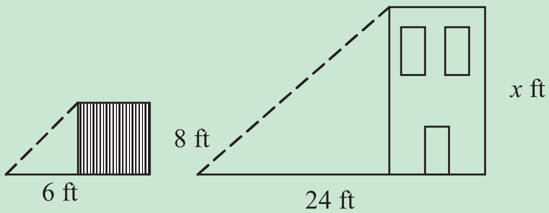
$$\frac{x}{21} = \frac{1.5}{2}$$

$$2x = 31.5$$

$$x = 15.75$$

The tree is 15.75 meters tall.

Draw right triangles for each object and shadow. The sun casts rays at the same angle, so the corresponding acute angles are congruent. The triangles are similar by AA Similarity.



Write and solve a proportion using ratios of corresponding sides.

$$\frac{6}{24} = \frac{8}{x}$$

$$\frac{1}{4} = \frac{8}{x}$$

$$\frac{1 \times 8}{4 \times 8} = \frac{8}{32}$$

The building is 32 feet tall.

Look Beyond

In the next module, students will extend their knowledge of similarity to include similar polygons. This means they will study similar rectangles, pentagons, hexagons, and so forth. The concept of scale factor will be introduced and will be used to solve problems about enlarging and reducing rectangular-shaped photographs.

Often times, indirect measurements can be found by using only one triangle. In fact, students will learn how to find a missing distance which involves one right triangle when they learn the Pythagorean Theorem in the next lesson.

Connections

Indirect measurement has been used to find the heights of very tall objects, including mountains. These methods have varied throughout the years to include scientific principles that analyze how barometric pressure changes with elevation as well as trigonometric calculations.

Distance is not the only measure which can be found indirectly. Weight and volume can also be measured indirectly.