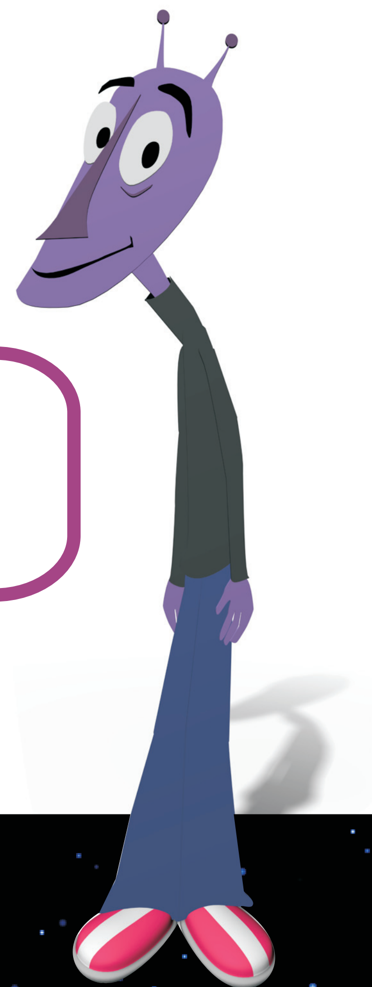
Geometry

★ Module 8 ★

Points, Lines, Angles, and Triangles



Lesson 5
Congruent Triangles

Teacher Notes 8.5

Objectives

- Identify congruent triangles and corresponding parts of congruent triangles.
- Model and identify the properties of congruent triangles.

Prerequisites

Naming triangles and identifying the parts of a triangle

Identifying congruent angles

Vocabulary

Equilateral triangle (8.4)
Congruent figures
Triangle (8.4)
Corresponding parts
Angle (8.1)
Vertex of an angle (8.1)
One-to-one correspondence
Congruence statement
Side-Side-Side Congruence
Side-Angle-Side Congruence
Included angle
Angle-Side-Angle Congruence
Included side
Right angle (8.2)
Bisect

Get Started

- Prior to class, prepare for a short demonstration by cutting thin strips of paper or cardboard. Cut four strips that are 12 inches long, one strip that is nine inches long, and one strip that is 15 inches long. Also bring clear or masking tape to class.
- Show students three 12 inch sides. Ask what type of triangle would be formed if the pieces were joined together at their endpoints.
  Possible answers: Acute triangle, equilateral triangle
- Join the three strips together to form an equilateral triangle. Tape the triangle to the board for all to see. Ask if there is a way to form a right triangle out of these sides if their endpoints always have to be taped together. **No**

- Show students the remaining three strips and ask what type of triangle they think it will form when the pieces are joined at their endpoints.
  Possible answers: Scalene triangle, right triangle

- Join the three strips together to form a scalene right triangle. Ask if there is a way for the sides to form a triangle that is not a right triangle if their endpoints always have to be taped together. **No** Discuss how the side lengths of a triangle determine the angle measures and, therefore, determine the shape of the triangle.

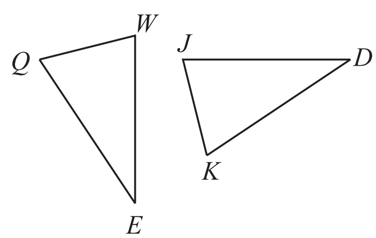
Subtopic 1 — Congruence

Expand Their Horizons

In this subtopic, students learn about congruent figures; in particular, they learn about congruent triangles. Congruent figures are any two figures with the exact same size and shape, as if one is a copy of the other. Similar to congruent angles, congruent figures do not need to be in the same orientation. One will fit precisely on top of the other, although one of the figures may need to be flipped or turned to make the matching parts align.

The sides and angles that “match up” are corresponding parts. Two congruent triangles have six corresponding parts: three corresponding angles and three corresponding sides. All six corresponding parts are congruent.

A congruence statement does more than indicate that two triangles are congruent. The order of the letters indicates which angles and sides are corresponding. This is especially important when the two triangles are not positioned the same way, such as with the triangles below.



Without a congruence statement, pairing up the shorter sides is not difficult, but it is difficult to tell how the two other sides pair up. With the congruence statement of $\triangle QWE \cong \triangle JKD$, we can see that since Q and J are both written first, that they are corresponding and congruent. W and K are both written in second place, so they are corresponding. Angles E and D are both written last, so they too are corresponding.

Students may choose to redraw one of the figures to help locate the matching sides although this information can also be obtained from the congruence statement alone. The endpoints of the corresponding segments are located in the same position. $\overline{QW}$ corresponds to $\overline{JK}$ because those endpoints are located first and second in the congruence statement: $\triangle QWE \cong \angle JKD$. Likewise, $\overline{QE}$ corresponds to $\overline{JD}$ because their endpoints are both first and third: $\triangle QWE \cong \triangle JKD$.

There is more than one correct congruence statement for two congruent triangles because the vertices in the first triangle can be written in any order. Once the first triangle is named, the vertices of the second triangle must be written in corresponding order. The following are correct congruence statements for the triangles above.

$$\triangle WEQ \cong \triangle KDJ \qquad \triangle EQW \cong \triangle DJK$$

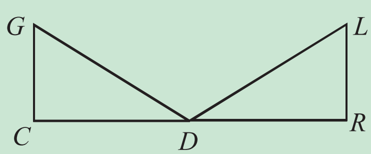
Notice that in every congruence statement, *W* is in the same position as *K*, *E* with *D*, and *Q* with *J*.

1 Once the shortest and longest sides have been identified, the second triangle can be redrawn. If it helps, rotate the triangles so that they are in the same position.

2 Write the first part of the congruence statement by listing the angles in the first triangle. Write the second part by listing the angles in the second triangle so that the corresponding angles are listed in the same order. The information about the sides can be used to verify that the congruence statement is correct because the endpoints will also be listed in the same order.

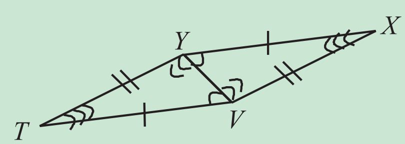
Additional Examples

1. In the figure below, $\triangle GDC \cong \triangle LDR$. Write another congruence statement for the triangles.



Write the vertices of the first triangle in another order: $\triangle CGD$.
Write the vertices of the second triangle so that they correspond with the first:
$\triangle CGD \cong \triangle RLD$.

2. For the triangles below, list the six corresponding parts and write a congruence statement.



Identify the congruent angles:
$\angle XYV \cong \angle YVT$, $\angle YVX \cong \angle VYT$, $\angle T \cong \angle X$.
Identify the congruent sides:
$\overline{VT} \cong \overline{YX}$, $\overline{YT} \cong \overline{VX}$, $\overline{YV} \cong \overline{YV}$.
Notice that the last pairs of sides is a common side:
$\triangle VYT \cong \triangle YVX$.

Subtopic 2 Determining Whether Triangles Are Congruent

Expand Their Horizons

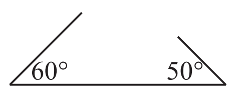
In this subtopic, students learn that they do not need to show that all six corresponding parts of two triangles are congruent to prove that the triangles are congruent. As demonstrated in the Get Started activity, with three sides of a given length, only one triangle can be formed. If the three sides of one triangle are congruent to three sides of another triangle, the triangles must be congruent. This is called the Side-Side-Side Congruence, which is often abbreviated as SSS Congruence.

A similar activity can be performed to demonstrate the Side-Angle-Side, or SAS Congruence. Tape two strips together at a given angle. There is only one possible length for the third side, and therefore, only one triangle to be formed.

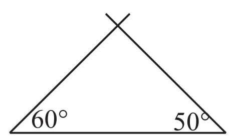
Common Error Alert:

Students may try to use Side-Angle-Side Congruence when the angle is not the included angle (not between the two sides). Point out that the word *angle* is written in the middle because the angle must be between those sides in the triangle. Read below for how to show that Side-Side-Angle does not prove congruence.

To demonstrate Angle-Side-Angle, or ASA Congruence, tape one strip to the board and draw two angles of a given measure. In this figure we used 50° and 60°.

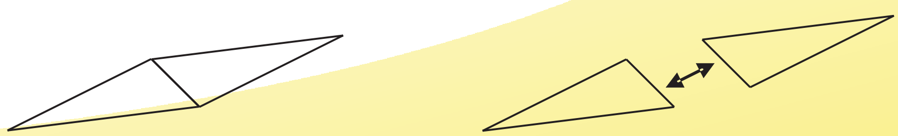


Extend the sides of the angles until they intersect. This is the only triangle that can be formed with the initial side length and two given angle measures.



On the contrary, Side-Side-Angle does *not* prove congruence because more than one triangle with two given side lengths and one angle that is not between the two sides can be formed. The triangles below have two pairs of congruent sides and one congruent angle. The lengths of third sides though are different. The triangles are not congruent.

Parts of congruent triangles may be congruent and not labeled. For instance, when triangles share a common side, the common side represents congruent sides because they are overlapping exactly. Show how the triangles below can be separated and how the common side becomes two congruent segments.



Remind students that vertical angles are always congruent, so they too may not be labeled as congruent.

3

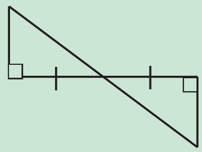
Look at the congruent parts. Two angles in one triangle are congruent to two angles in another. The congruent side is the included side between those two angles. The triangles are congruent by the Angle-Side-Angle Congruence.

4

The three congruent parts are three sides. The triangles are congruent by the Side-Side-Side Congruence.

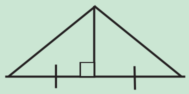
Additional Examples

1. Determine if the triangles are congruent. If so, state why.



The right angles are congruent because they both measure 90°. The vertical angles are congruent. The triangles are congruent by ASA Congruence.

2. Determine if the triangles are congruent. If so, state why.



The triangles share a common side. That creates a second pair of congruent sides. The included angles are both right angles. One of the right angles is not labeled, but because they form a line, which is a straight angle with a measure of 180°, the other angle must also measure 90°. The triangles are congruent by SAS Congruence.

Look Beyond

Congruent triangles are the exact same size and shape. Students will next learn about similar triangles, which are the same shape, but not the necessarily the same size. Similar to proving triangles congruent, there will be shortcut rules to prove that two triangles are similar, such as Angle-Angle Similarity, or AA Similarity. They will also form similarity statements by listing the vertices in corresponding order. This is analogous to a congruence statement except the symbol for similar (~) is used.

Connections

Congruent figures, when positioned symmetrically or in a repeating pattern, can create striking and captivating works of art, which is why they are used abundantly in the world of arts and crafts. For example, designs for blankets, quilts, and pottery dishes often include congruent figures.