Geometry
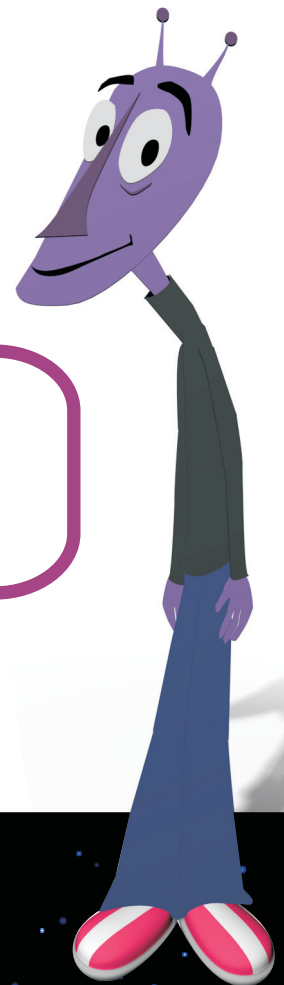
★ Module 8 ★

Points, Lines, Angles, and Triangles

Lesson 4
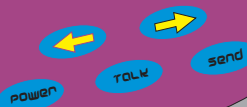Triangles

Objectives

♦ Identify, describe, draw, and classify triangles as equilateral, isosceles, or scalene.

♦ Identify, describe, draw, and classify triangles as right, acute, obtuse, and equiangular.

♦ Use physical models and paper to determine the sum of the measures of interior angles of triangles.

POWER   TALK   send

Prerequisites

Classifying angles and estimating angle measurements

Naming angles and line segments

Vocabulary

Triangle
Line segment (8.1)
Noncollinear (8.1)
Vertex of a triangle
Angle (8.1)
Right angle (8.2)
Right triangle
Obtuse triangle
Acute triangle
Acute angle (8.2)
Obtuse angle (8.2)
Equilateral triangle
Congruent (8.2)
Isosceles triangle
Scalene triangle
Line (8.1)
Triangle Sum Property
Equiangular triangle

Get Started

• Have each student draw and cut a triangle out of a piece of paper. Tell students to draw a triangle of any shape or size that they want, or have a variety of triangles ready before class and pass them out so each student has one triangle.

• Have students hold their triangle so that one side of the triangle is parallel to the top of their desk. Then, have students fold the top part of the triangle down so that the top corner touches the bottom side and have them bring the two bottom sides together so that those corners are on the same point as the first. These three angles, which were the corners of a triangle, form a straight line—regardless of the original size or shape of the triangle. In this lesson students will discover why the three corners form a straight line.

8.4A

Expand Their Horizons

In this subtopic, students learn the formal definition of a triangle. It is a closed plane figure made up of three line segments that intersect only at their endpoints. The line segments are called the sides, and the endpoints are the vertices. Explain that the word, *vertices,* is plural for vertex.

Ask students to name other words that begin with *tri-*. Possible answers are tricycle and triathlon. Point out that the prefix *tri-* means *three*. The name triangle literally means *three angles*.

A triangle is named by writing a small triangle symbol followed by the vertices. The vertices can be written in any order. A triangle with vertices *F*, *G*, and *H* can be named $\triangle FGH$, $\triangle FHG$, $\triangle GFH$, $\triangle GHF$, $\triangle HFG$, or $\triangle HGF$.

1 The sides are the three line segments that form the triangle. The vertices are the endpoints of the segments. There is one angle at each vertex. Name the triangle by writing the vertices in any order.

Common Error Alert:

Students may forget to, or choose not to, write $\triangle$ before the three letters when naming a triangle. Point out, that without the triangle symbol, the geometric figure could be taken for a plane or an angle. Since planes, angles, and triangles can all be named by three points, it is critical that they be preceded by the correct word or symbol.

2 List the three line segments as the sides and the three endpoints as the vertices. One angle is at each vertex. Name the triangle by using all three vertices in any order.
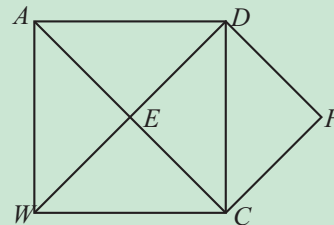
Additional Examples

1. **Explain why the following figure is not a triangle.**
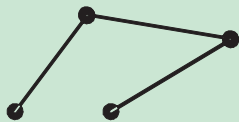
   The figure is made up of three line segments, but the figure is not closed. The sides do not all meet at their endpoints.

2. **Name all the triangles that have point *E* as one of its vertices.**

   Identify the triangles with *E* as an endpoint of one of its sides.

   $\triangle AEW$, $\triangle AED$, $\triangle DEC$, $\triangle CEW$

Expand Their Horizons

In this subtopic, students classify triangles both by their angle measures and by their sides. Every triangle has at least two acute angles. It is the measure of third angle that determines the angle's classification. If the third angle is a right angle, the triangle is a right triangle. If the third angle is an acute angle, the triangle is an acute triangle. In other words, all three angles in an acute triangle are acute. If the third angle is an obtuse angle, the triangle is obtuse.

A triangle can also be classified according to how many of its sides are congruent. If none of the sides are congruent, the triangle is a scalene triangle. If at least two of the sides are congruent, the triangle is isosceles. If all three sides are congruent, the triangle is equilateral. By definition, an equilateral triangle is also isosceles.

The congruent sides of an isosceles triangle are called the legs. The non-congruent side is called the base. Point out that if two sides of a triangle are congruent, then two angles (the angles across from the legs) are also congruent.

Students will learn later in this module that the side opposite the right angle in a right triangle is called the hypotenuse. The other two sides are the legs. The hypotenuse of any right triangle is greater than either of its legs. It is therefore impossible for a triangle to be both equilateral and right.

3 The triangle has two congruent sides which are marked as such. The angle measures are not given, but each is less than a "corner" which is 90°. It is an acute isosceles triangle.

4 The left bottom angle is an obtuse angle. None of the sides are labeled as congruent. It is an obtuse scalene triangle.

5 To draw an obtuse isosceles triangle, draw one obtuse angle. Add a third side. Adjust the sides as needed so that two of them are congruent. To draw a right scalene triangle, draw a right angle with sides of different lengths. Add the third side. The third side will be greater than either of the legs, so all three sides are non-congruent. To draw an acute scalene triangle, draw a triangle with all angles less than 90°. Be sure that none of the sides have the same length.

Additional Examples

1. **The measures of a triangle are 40°, 40°, and 100°. Classify the triangle by its angles.**

   One of the angles is obtuse because its measure is greater than 90°. It is an obtuse triangle.

2. **The sides of a triangle measure 10 inches, 10 inches, and 15 inches. Classify the triangle by its sides.**

   Two of the sides are congruent. The triangle is isosceles.

8.4C

Expand Their Horizons

In this subtopic, students learn that the sum of the measures of any triangle is 180°. This is called the Triangle Sum Property. Refer students back to the Get Started activity. For every student's triangle, the three corners formed a straight line. A straight line measures 180°, so the three angle measures have a sum of 180°.

Students also learn in this subtopic that a triangle with three congruent measures is an equiangular triangle. Because 180° ÷ 3 = 60°, the measure of each angle of an equiangular triangle is always 60°. Because congruent angles occur opposite congruent sides, an equiangular triangle is also equilateral.

6   Add the two given measures: 58° + 82° = 140°. Subtract the sum from 180°. 180° − 140° = 40°. The third interior angle measures 40°.

7   One of the angles is a right angle, which measures 90°. Because 180° − 90° = 90°, the missing angle measure can be found by subtracting 68° from 90°: 90° − 68° = 22°. The gangway forms a 22° angle with the ground.

8   Find the sum of each set of angles. The second set does not sum to 180°, so that set does not form a triangle. The first and third sets do. The first set has a right angle, so the triangle is a right triangle. The second set has all acute angles, so the triangle is an acute triangle.
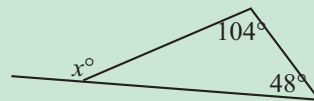
Additional Examples

1. **Two angles of a triangle measure 40° and 30°. Classify the triangle by its angles.**

   The given angles have a sum of 70°.

   Subtract this sum from 180°:
   180° − 70° = 110°.

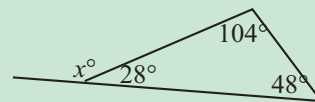   Because the third angle is greater than 90°, the triangle is an obtuse triangle.

2. **Find the value of x.**

   First find the measure of the third interior angle.
   104° + 48° = 152°
   180° − 152° = 28°

   Because they form a line, $x° + 28° = 180°$.
   180° − 28° = 152°.

   The value of x is 152.

Look Beyond

Students will learn about polygons in the next module. A polygon is a closed plane figure made up of line segments joined only at their endpoints. A triangle is the smallest possible polygon. There is no limit to the number of sides in a polygon.

A polygon with four sides is a quadrilateral. Students will investigate and learn the Quadrilateral Sum Property, which gives the measures of the interior angles of a quadrilateral. In more advanced courses, they will derive a formula to find the sum of the interior angles of any polygons.

Connections

Because the triangle is the strongest, most stable of all the polygons, it is used in many buildings, especially in the framework. A triangle cannot be bent. Notice how a rectangle can be bent over and deformed.
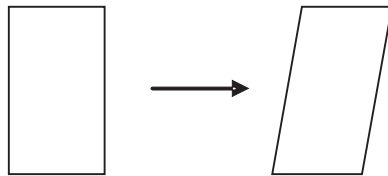
This bending can be caused by weight and pressure from surrounding beams. If bent enough, the rectangle will break. Bridge designs utilize triangles because bridges undergo a great amount of stress from both man and the environment. Even smaller structures, such as a tall shelf or fence gate, may require diagonal bracing along the back.