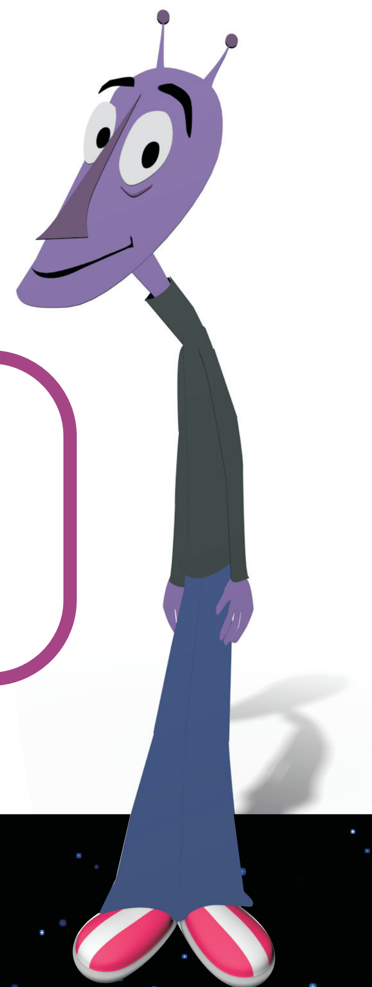
Geometry

Module 8

Points, Lines, Angles, and Triangles



Lesson 3
Angle Relationships and Parallel Lines

Teacher Notes 8.3

Objective

- Recognize the pairs of angles formed and the relationship between the angles including two intersecting lines and parallel lines cut by a transversal (vertical, supplementary, complementary, corresponding, alternate interior, alternate exterior angles, and linear pair).

Vocabulary

Angle (8.1)
Complementary angles
Supplementary angles
Right angle (8.2)
Straight angle (8.2)
Intersecting lines (8.2)
Vertical angles
Congruent angles (8.2)
Transversal
Coplanar (8.1)
Corresponding angles
Alternate interior angles
Alternate exterior angles
Parallel lines (8.2)

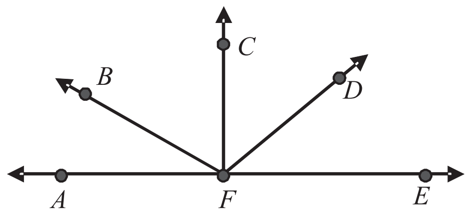
Prerequisites

Identifying right angles and straight angles

Identifying intersecting lines and parallel lines

Get Started

- Draw the following diagram on the board.



- Challenge students to list all the angles in the diagram.
  ∠AFB, ∠AFC, ∠AFD, ∠AFE, ∠BFC, ∠BFD, ∠BFE, ∠CFD, ∠CFE, ∠DFE

- Ask students which angles appear to be right angles. ***∠AFC, ∠CFE, ∠BFD***

- Ask students which angles, when put together, appear to form right angles. **∠*AFB* and ∠*BFC*, ∠*CFD* and ∠*DFE*, ∠*BFC* and ∠*CFD***

- Have students name combinations of angles that, when put together, appear to form a straight angle. **Possible answers: ∠*AFC* and ∠*CFE*; ∠*AFB*, ∠*BFC*, ∠*CFD*, and ∠*DFE*; ∠*AFC*, ∠*CFD*, and ∠*DFE***

- Tell students in today's lesson they will be combining angles as they learn about angle relationships.

Subtopic 1 — Angle Relationships

Expand Their Horizons

In this subtopic, students learn about complementary and supplementary angles. Complementary angles are two angles whose measures have a sum of 90°. It can also be defined as two angles whose measures *sum* to 90°, as sum can be used as both a noun and a verb. Complementary angles may or may not share a side. An angle greater than or equal to 90° has no complement.

Supplementary angles are two angles whose measures have a sum of 180°. Like complementary angles, supplementary angles may share a side. When they do, they form a straight angle or line. Two supplementary angles that form a line can be called a linear pair. Although this term is not defined in the lesson, teachers may wish to share it with students because many common geometric figures include linear pairs.

Common Error Alert:

Complementary and supplementary angles are always *pairs* of angles. Students may call a single right angle a complementary angle, or a single straight angle a supplementary angle. Stress, that although the measures are 90° and 180°, that they are not formed by *two* angles and cannot be identified as complementary or supplementary.

Some students may have difficulty remembering which name goes with which pair. Share that some students remember it by thinking that C comes before S in the alphabet, and 90 comes before 180 on a number line. Teachers can also point out that the letter C forms the top of the 9 in 90° and the letter S almost forms the 8 in 180°.

C 0°   1 S0°

1 Look for two angles whose sum is 90°. ∠*E* can be eliminated from the choices because it is already greater than 90°. The pairs are ∠*A* and ∠*D* and ∠*B* and ∠*C*.

2 Look for two angles whose sum is 180°. The pairs are ∠ *A* and ∠*C* and ∠*B* and ∠*E*.

Additional Examples

1. **Find the measure of the complement and supplement of an angle that measures 20°.**

   To find the complement, subtract from 90°: 90° – 20° = 70°.

   To find the supplement, subtract from 180°: 180° – 20° = 160°.

   The complement measures 70°, and the supplement measures 160°.

2. **Find the measure of the complement and supplement of an angle that measures 170°.**

   The angle has no complement because its measure alone is greater than 90°.

   To find the supplement, subtract from 180°: 180° – 170° = 10°.

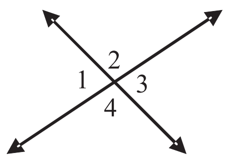
   The supplement measures 10°.

Subtopic 2

Intersecting Lines and Transversals

Expand Their Horizons

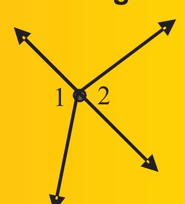
In this subtopic, students learn about vertical angles and angles formed by two lines and a transversal. Vertical angles are the nonadjacent angles formed by two intersecting lines. In other words, when two lines intersect, they are the angles that do not share a side. In the figure below, $\angle 1$ and $\angle 3$ form one pair of vertical angles, and $\angle 2$ and $\angle 4$ form the other pair.



Vertical angles are congruent. This can be proved by using the fact that the measures of two angles that form a line sum to 180°. In the figure above, $m\angle 1 + m\angle 2 = 180°$. Likewise, $m\angle 2 + m\angle 3 = 180°$. Therefore, $m\angle 1 = m\angle 3$. This conclusion can be proved by either substituting measures for angles 1, 2, and 3, or by using algebra and substituting $m\angle 2 + m\angle 3$ for 180° in the first statement: $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$. When $m\angle 2$ is subtracted from both sides, $m\angle 1 = m\angle 3$.

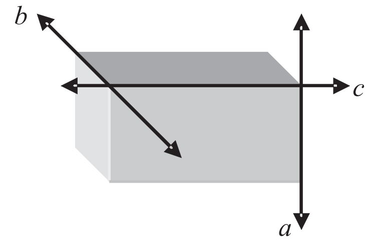
Common Error Alert:

Students may say that $\angle 1$ and $\angle 2$ in the figure below are vertical angles.

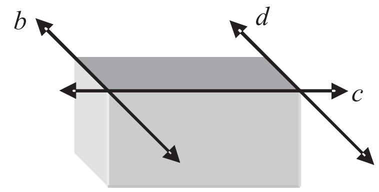


Point out that while they are angles that share a vertex and have no common sides, the angles are not formed by two lines. A line cannot make a turn.

Next, students learn the terms used when two lines are intersected by another line, which is called a transversal. A transversal intersects two or more coplanar lines. Show students the following diagram to demonstrate what a line intersecting two noncoplanar lines would look like.

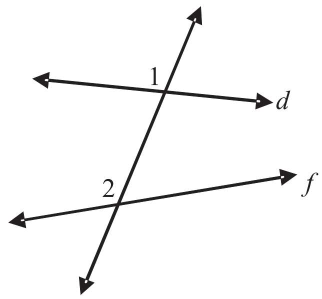


Line *c* intersects both lines *a* and *b*. However, line *c* is not a transversal through lines *a* and *b* because *a* and *b* do not lie in the same plane.

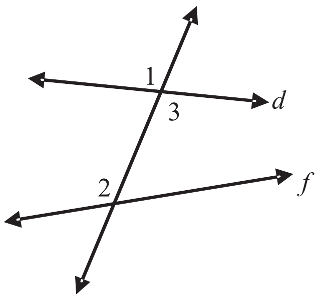


On the other hand, line *c* is a transversal through lines *b* and *d* because *b* and *d* both lie in the top plane of the box.

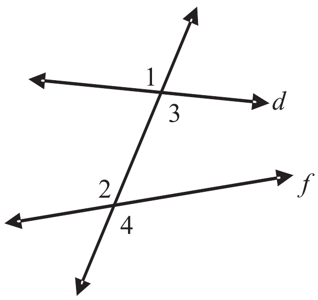
When a transversal intersects, or cuts, two lines, eight angles are formed, four with one line and four with the other. The angles that are in the same position in each group (e.g. top left, bottom right) are called corresponding angles. In the figure below, $\angle 1$ and $\angle 2$ form a pair of corresponding angles because they are both in the top left corner in their group of four.



The four angles that are "in between" the two lines that are not the transversal are interior angles. If they are in separate groups of four and are on opposite sides of the transversal, they are called alternate interior angles. In the figure below, ∠2 and ∠3 are alternate interior angles.



The four angles that are not in between the two lines are exterior angles. If they are in separate groups of four and are on opposite sides of the transversal, they are called alternate exterior angles. In the figure below, ∠1 and ∠4 are alternate exterior angles.

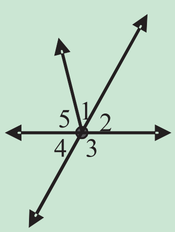


3 ∠2 and ∠8 are not between lines *m* and *n*. They are exterior angles. Since they lie on opposite sides of the transversal, they are alternate exterior angles. ∠1 and ∠6 are both in the bottom right corner of their group, so they are corresponding angles. ∠3 and ∠5 are in between lines *m* and *n*. They are interior angles. Since they lie on opposite sides of the transversal and are in different groups, they are alternate interior angles.

Some students may refer to alternate interior angles as "diagonal inside" and alternate exterior angles as "diagonal outside."

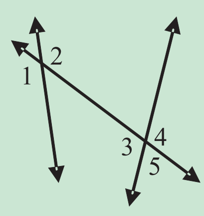
Additional Examples

1. Identify the vertical angles below.



2. Identify the special angle pair name for each pair below. If none, write *none*.

∠1 and ∠3
∠2 and ∠5
∠2 and ∠3
∠1 and ∠5
∠1 and ∠4
∠2 and ∠4



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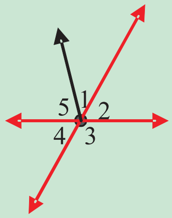
Vertical angles are formed by two lines.



The only pair of vertical angles in the diagram is ∠2 and ∠4.

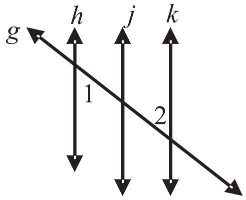
∠1 and ∠3: Both bottom left in their group: Corresponding
∠2 and ∠5: One is interior and one is exterior: None
∠2 and ∠3: Different groups, different sides of transversal, both interior: Alternate interior
∠1 and ∠5: Both exterior, but same side of transversal: None
∠1 and ∠4: Different groups, different sides of transversal, both exterior: Alternate exterior
∠2 and ∠4: Both top right in their group: Corresponding

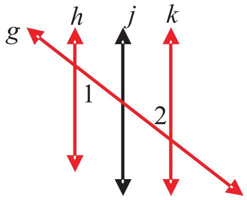
Subtopic 3 Parallel Lines and Transversals

Expand Their Horizons

In this subtopic, students learn that if the two lines cut by a transversal are parallel, the corresponding, alternate interior, and alternate exterior angles are congruent. Have students use the lines provided on notebook paper to draw two parallel lines. Then, have them draw a transversal intersecting them at any angle. Have students measure the angles with a protractor to confirm the rules.

Students may become confused when a transversal cuts more than two lines. In the figure below, line *g* is a transversal that intersects three parallel lines. To determine *m*∠2, given that *m*∠1 = 50°, ignore line *j*. It may help to highlight lines *h* and *k* and the transversal *g*.





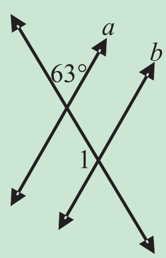
∠1 and ∠2 are alternate interior angles. Because alternate interior angles are congruent when the lines are parallel, *m*∠2 = 50°.

When parallel lines are cut by a transversal, the measure of any angle can be found when given the measure of any other angle by using any of the rules about supplementary, vertical, corresponding, alternate interior, or alternate exterior angles.

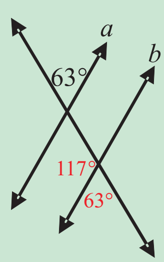
4 The angles can be found in any order with any combination of rules. Because ∠1 is vertical to the angle whose measure is given as 125°, $m\angle 1 = 125°$. There are two reasons why $m\angle 8 = 125°$. ∠8 and the original angle labeled 125° are corresponding, and ∠8 and ∠1 are alternate interior. Because ∠7 and ∠8 form a line, their measures sum to 180°, making $m\angle 7 = 55°$.

Additional Examples

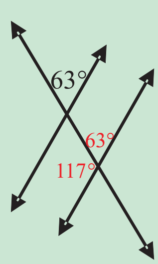
1. Explain how to find $m\angle 1$.



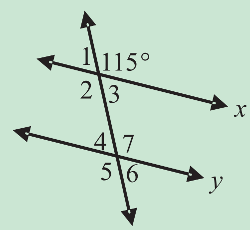
One way is to label the alternate exterior angle 63° and then find its supplement.



Another way is to label the corresponding angle 63° and find its supplement.



2. Which angles have a measure of 115°? Why?



∠2 and the given angle are vertical angles.

∠7 and the given angle are corresponding angles.

∠5 and the given angle are alternate exterior angles.

∠2, ∠7, and ∠5 all have a measure of 115°.

Look Beyond

In more advanced geometry studies, students will learn that interior angles in different groups, but on the same side of the transversal, are called either same-side interior or consecutive interior angles.

The measures of angles formed by parallel lines can be used to solve problems in later lessons of geometry, such as in the study of quadrilaterals. Parallelograms and trapezoids have at least one pair of parallel lines, and the sides of those figures can be extended to show alternate interior and consecutive interior angles. Medians (line segments formed by connecting the midpoints of sides) of triangles are parallel to a side of the original figure. Therefore, the rules in this lesson can be used to find missing angle measures in those figures as well.

Connections

Interior designers can use the properties of parallel lines to be sure rows of picture frames or other art pieces are parallel to each other. The distance between two parallel lines never changes, given that the distance is measured at a right angle. Suppose a designer needs to hang six art pieces in two rows of three pieces each. He or she can hang the first piece and then measure the distance from the ceiling to the top of the first piece. To find where to vertically hang the next piece, he or she measures that same distance from the ceiling. To hang the second row, the designer can measure either from the ceiling to the first piece in the second row, or from an art piece in the top row, if they are to be hung directly above and below each other.