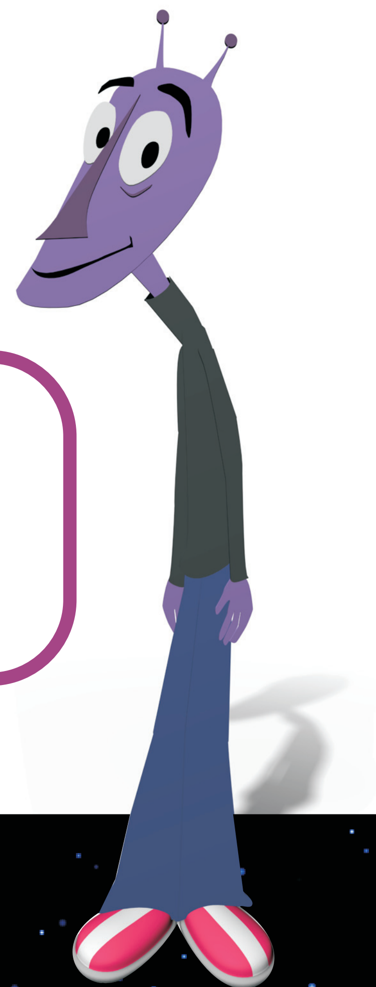
Geometry

★ Module 8 ★

Points, Lines, Angles, and Triangles



Lesson 2
Angle Classifications and Line Relationships

Teacher Notes 8.2

Objectives

- Identify parallel, perpendicular, and intersecting lines.
- Identify, draw, and measure congruent, adjacent, obtuse, acute, right, and straight angles.
- Use benchmark angles to estimate the measure of angles (e.g. 45 degrees, 90 degrees, 120 degrees, and 180 degrees).

POWER TALK SEND

Prerequisites

Identifying and naming angles

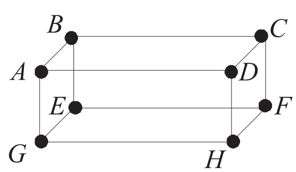
Identifying and describing points, rays, lines, and planes

Vocabulary

Angle (8.1)
Infinite (3.1)
Degree
Protractor
Ray (8.1)
Endpoint (8.1)
Right angle
Straight angle
Line (8.1)
Acute angle
Obtuse angle
Benchmark angle
Congruent
Congruent angles
Intersecting lines
Perpendicular lines
Parallel lines
Plane (8.1)
Point (8.1)

Get Started

- Draw the diagram on the board and have students answer the following:



- How many line segments are in the figure? **12**
- Name three planes that contain point *C*. **Plane *ABC*, plane *EBC*, plane *DCF***
- What point is coplanar with points *A*, *G*, and *E*? **Point *B***

- Name three segments that all contain point H. **$\overline{GH}$, $\overline{DH}$, and $\overline{FH}$**

- Name a segment that is coplanar with $\overline{AD}$ and $\overline{DH}$. **$\overline{AG}$ or $\overline{GH}$**

- Tell students that being able to identify points, line segments, and planes will be necessary for today's lesson on angle classification and line relationships.

Subtopic 1 — Angle Classification

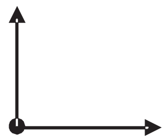
Expand Their Horizons

In this subtopic, students first learn how to use a protractor to draw and to measure angles. The measure of an angle indicates its *degree of openness*, which is how far apart the two sides of the angle are from each other. It does not measure the lengths of the sides of the angle because the sides are rays and can be extended indefinitely.

In this lesson, students measure the interior of an angle which has a measure greater than 0° and less than or equal to 180°. An angle cannot measure 0° because that would mean that one ray was lying on top of the other, which is indistinguishable from a single ray.

Recall that the definition of an angle is two *distinct* rays joined at an endpoint. Rays in this position are not distinct.

To form an angle that measures 180°, the rays must be facing in opposite directions, making a line. A 90° angle is halfway between 0° and 180°.



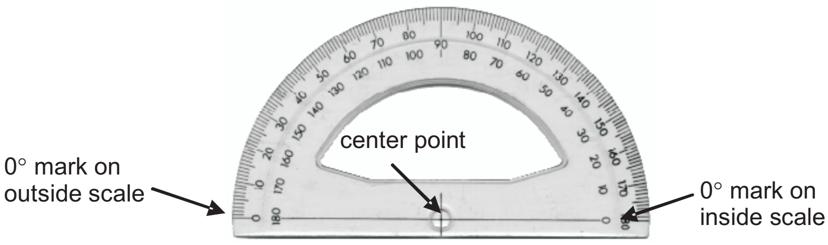
90°

180°

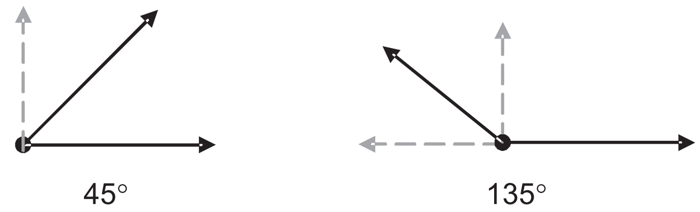
To draw an angle of a given measure, first draw one ray. This is one side of the angle. It does not matter which direction the ray is facing, but a horizontal ray facing to the right is a common way to start. Then, place a protractor on the ray so that the center point, often a small open circle, is on the endpoint of the ray. Align the ray with the 0° mark. If there are two 0° marks, the ray should be facing the one it passes through. The ray can be extended to be sure it is properly aligned. On the same scale as this 0° mark, locate the given measure and place a dot. Last, draw the second ray from the endpoint of the first ray through this point.

Common Error Alert:

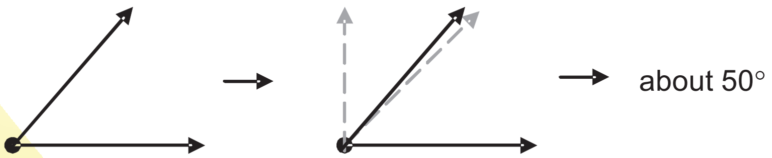
Students often use the wrong scale on a protractor. The 0° mark may be on the "inside" or "outside" scale. Tell students to follow that same scale until they reach the measure they want. They can trace the scale they need from 0° with their pencil so that they do not accidentally read the wrong number.



Angles that measure 90° and 180° are called benchmark angles because their degree measure is easily recognizable, based on their shape. Two other easily recognizable angles are the angles halfway to a 90° angle, which is a 45° angle and halfway between 90° and 180°, which is a 135° angle.



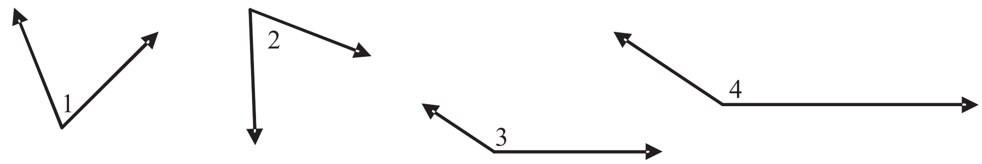
Benchmark angles can be used to estimate the measures of other angles. For example, to estimate the measure of the angle below, sketch a 90° angle and then divide it in half. The degree of openness is just greater than 45°, so the angle measure is about 50°.



Other benchmarks angles can be found by dividing the angle-measure range into thirds: 30°, 60°, 120°, and 150°.

Angles are classified according to their measure. Angles between 0° and 90° are acute angles. Angles between 90° and 180° are obtuse angles. Angles that measure 90° are right angles, and angles that measure 180° are straight angles. A straight angle is also a line because it is a set of points that extends in opposite directions without end. Point out that most corners are right angles, such as the corner of a room or a piece of paper.

Angles with the same measure are called congruent angles. Angles do not need to be facing the same direction to be congruent. Remind students also that the lengths of the sides are irrelevant, because both rays can be extended indefinitely. In the figure below, $\angle 1$ and $\angle 2$ are congruent, and $\angle 3$ and $\angle 4$ are congruent. All right angles are congruent.



1 The angle measure is about one-third the measure of a right angle, so the measure is about 30°. This is between 0° and 90°, so the angle is acute.

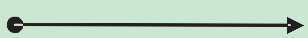
2 The angle is obtuse because its degree of openness is greater than a right angle. It is about one-third of the way between a right angle and a straight angle, so the measure is about 120°. Students can place the corner of an index card inside the angle to see that the angle measure is greater than 90° because a corner fits inside of it.

3 The angles are not congruent because the degree of openness is different. The angle on the right is more open than the one on the left.

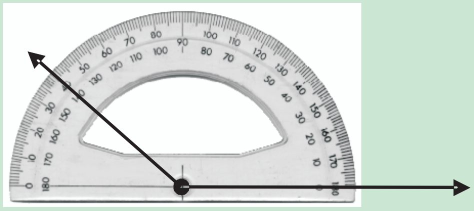
Additional Examples

1. Use a protractor to draw an angle that measures 140°.

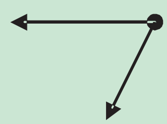
Draw a ray.



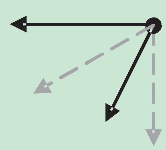
Place a protractor so that the small open circle or center point is on the endpoint of the ray and align the ray with the 0° mark. Locate 140° on the protractor. Draw the second ray to that point.



2. Classify the angle and estimate its measure.



The angle measure is about two-thirds the measure of a right angle.



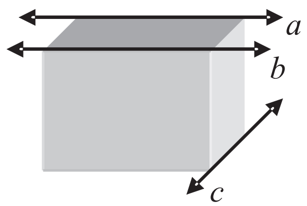
The angle is about 60°. It is an acute angle because the measure is between 0° and 90°.

Expand Their Horizons

In this subtopic, students learn about intersecting and nonintersecting lines. Intersecting lines have a point in common, called their point of intersection. In everyday terms, the lines “cross each other.” If the angles formed by the intersection are right angles, the lines are said to be perpendicular.

The symbol for perpendicular is an upside T shape: $\perp$. Show how the symbol is formed by two perpendicular segments. Rays can also be perpendicular. Show students pictures of a ray perpendicular to a line, a segment perpendicular to a ray, and so forth. In these cases, the figures may not “cross each other,” but they do “touch,” which means they have a point in common.

Parallel lines are coplanar lines that do not intersect. It is important to mention that the lines are in the same plane because it is possible for nonparallel lines to be nonintersecting. In the figure below, lines *a* and *b* are parallel because they are in the same plane and will never intersect. Lines *a* and *c* are not parallel. However, they will never intersect because they are in different planes. Lines that are neither parallel nor intersecting are said to be skew.



The symbol for parallel is two vertical parallel segments. In the diagram above, $a \parallel b$. As with perpendicular figures, rays and segments can also be parallel.

Planes can also be intersecting or parallel. In the figure above, the top and bottom planes are parallel. The top and right planes are intersecting. While intersecting lines have a common point, intersecting planes have a common line. The top and front planes in the above figure have line *b* in common.

Common Error Alert:

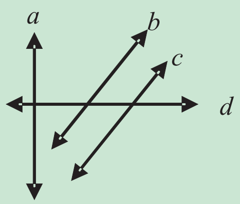
Remind students that lines and rays continue indefinitely. Students may say the lines below do not intersect, but it is apparent that they do once they are extended.

4 The lines have a point in common and form right angles. They are intersecting and perpendicular.

5 The lines intersect once they are extended. However, the angles formed are not right angles. They are not perpendicular.

Additional Examples

1. Tell which pairs of lines are perpendicular, which are intersecting and not perpendicular, and which are parallel.



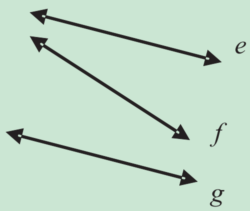
Lines *a* and *d* are perpendicular because they intersect to form right angles.

The following pairs are intersecting, but not perpendicular because they have a point in common but do not form right angles:

lines *a* and *b*
lines *a* and *c*
lines *b* and *d*
lines *c* and *d*.

Lines *b* and *c* are parallel because they will never intersect.

2. Tell which pairs of lines are perpendicular, which are intersecting and not perpendicular, and which are parallel.



None of the lines are perpendicular because none will intersect at a right angle.

Lines *e* and *f* and lines *f* and *g* are pairs of intersecting lines. They are not perpendicular.

Lines *e* and *g* are parallel. They will never intersect.

Look Beyond

In more advanced geometry classes, students will learn geometries that are different from the geometry they are now studying, which is called Euclidean geometry. Euclidean geometry is based on the premise that for a given line and point not on the line, there is exactly one line that passes through that point and is parallel to that line.

These other geometries include spherical geometry, where a line is defined as a circle on a sphere that contains a diameter of the sphere (called a great circle). Parallel lines do not exist in spherical geometry because great circles always intersect.

Another type of geometry is hyperbolic geometry in which a line is defined as an arc on a disk whose endpoints lie on the edge of the disk. In this case, several arcs can pass through a given point on one side of the disk and not intersect another arc on the other side. Therefore, in hyperbolic geometry, given a line and a point not on the line, there can be several lines that pass through that point that are parallel to that line.

Connections

Airplane pilots and ship captains need to understand angles in order to navigate. For example, the path of a flight might be described as 62° west of north. The captain of a ship may need to change his direction by 5° to miss an iceberg or stay out of the path of another ship.

Forest rangers also use angles to navigate through a forest or to give directions. If they spot a fire from a watchtower, they may report the fire in terms of its location from their tower giving its bearing (clockwise angle from north) and approximate distance from the tower. In these cases, angle measures extend up through 360°, where 90° is east, 180° is south, 270° is west, and 360° is north.