Geometry

★ Module 8 ★

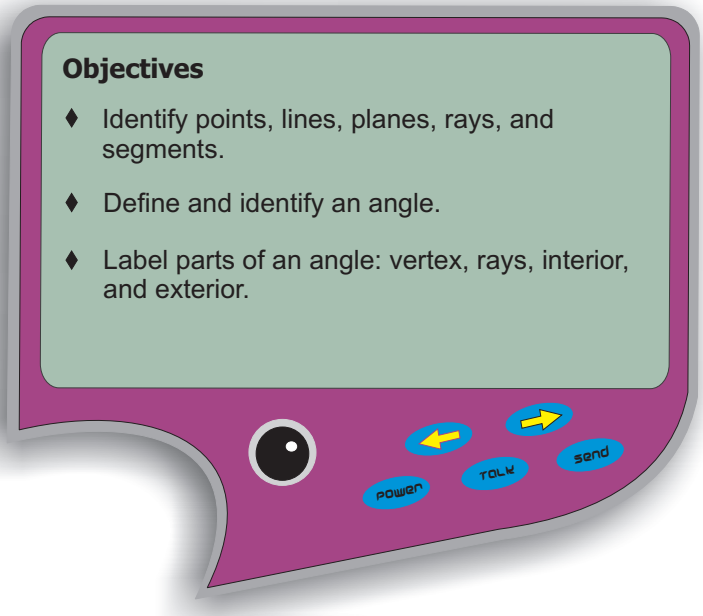
Points, Lines, Angles, and Triangles

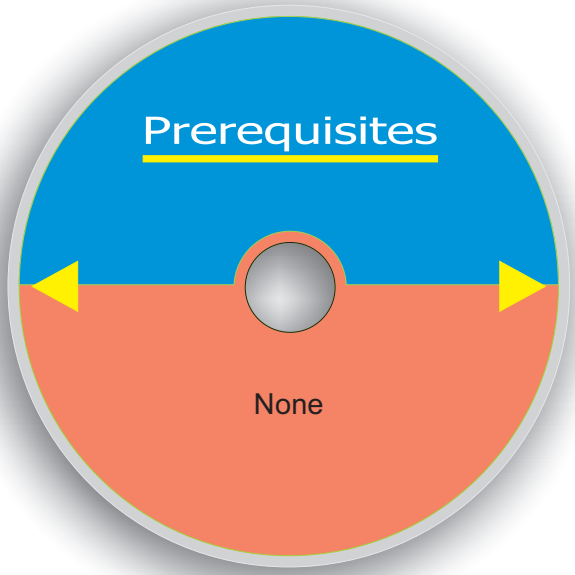
Lesson 1
Language of Geometry

Teacher Notes 8.1

Objectives

- Identify points, lines, planes, rays, and segments.
- Define and identify an angle.
- Label parts of an angle: vertex, rays, interior, and exterior.





Get Started

- Have students look around the classroom and make a list of where they see lines.
  Possible answers: The sides of windows, in the floorboards, the sides of a chalkboard, the edges of their desks

- Have students look around and make a list of where they see "flat surfaces."
  Possible answers: The floor, the walls, the ceiling, table tops, desk tops, tops of books

- Have students look around and make a list of where they see angles. If they are unfamiliar with this term, ask where they see "corners and turns."
  Possible answers: Corner of a room, corner of a book, the hands on a clock

- Tell students they are about to study geometry, a branch of mathematics that studies the properties of sets of points which form geometric figures such as lines, planes (flat surfaces), and angles. They will learn the formal definition of a line and will see that a line must be straight and will go on forever, so some of what they have listed as lines may not be considered lines in geometry, although they may be parts of lines.

Subtopic 1 — Basic Terms of Geometry

Expand Their Horizons

In this subtopic, students learn some basic figures in geometry and how to name them. The simplest figure is the point. A point represents a location and may be used to stand for an object that has a size, although the point itself actually has no size. It is represented by a dot and named by one uppercase letter.

All geometric figures are a collection of points. If the points form a straight path and go on forever in opposite directions, the figure is a line. Stress to students that in geometry, a line means a straight line. A curved figure, even if it goes on indefinitely, is not a line. Because we cannot draw a line that goes on forever, the picture of a line must have arrows at each end.

A line is named by writing any two points on the line, in any order, and drawing a line symbol over them. Two possible names for a line that contains points *A*, *B*, and *C* are $\overleftrightarrow{AB}$ and $\overleftrightarrow{CA}$. A line may also be named by one lowercase letter which may appear near the end of the picture of a line. Points on the same line are collinear. Any two points are collinear because one line can be drawn through any two points.

A plane can be thought of as a flat surface that extends forever in all directions. It is impossible to draw a figure that extends forever, so a plane is represented by a four-sided figure, drawn at an angle, for the purpose of giving it perspective. Make sure students understand that this is just a picture of a plane, and that a plane really has no edges.

A plane is named by three noncollinear points in the plane. They must be noncollinear points because an infinite number of planes contain any two points. A plane can also be named by one capital script letter, which might appear near one of the “corners” of the picture. This letter does not represent a point; it names the plane.

Points and lines on the same plane are coplanar. Any three points are coplanar because one plane can be drawn that contains those points. To help students comprehend this, have students imagine the floor and ceiling as two planes. Two points on the floor and one point on the ceiling may not seem to be coplanar, but there is an “invisible” plane that is vertical and/or diagonal to the floor and ceiling that contain them. Likewise, points on opposite walls are coplanar because of the invisible horizontal plane that contains them. Three noncollinear points on the floor and one point on the ceiling, however, are noncoplanar. No plane can possibly contain all four of those points.

A line segment is part of a line. The first and last points are called the endpoints. Often, a line segment is simply called a segment. It is named by its endpoints; it does not matter which endpoint is written first. A bar (a line without arrows) is drawn over those two letters. Students will later learn that by omitting the bar, they are referring to the length of the segment, rather than the segment itself.

A segment that goes on forever in one direction is a ray. Therefore, a ray has one endpoint. To name a ray, first write the endpoint and then any other point on the ray. A bar with an arrow on the right end is written over the two letters.

Common Error Alert:

Students may draw the ray symbol backwards when the picture of the ray extends from right to left. In this case, they may also write the endpoint as the second letter.

Incorrect: $\overrightarrow{BD}, \overleftarrow{BD}, \overleftarrow{DB}$ Correct: $\overrightarrow{DB}$

Tell students that the symbol always has the arrow on the right and the endpoint is always written first, regardless of which way the ray is facing.

1. Because only two points are labeled, those two points must be used to name the line. It does not matter in which order they are written.

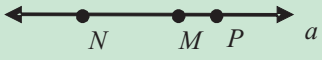
2. Points *A*, *B*, and *C* are noncollinear, so the plane can be named using those three points. The plane can also be named by the uppercase script letter *M*.

3. A point has no size, so it cannot be measured. The dot representing the point has thickness, but the actual point itself does not.

4. A line goes on forever, so it has an infinite number of points, not just two.

Additional Examples

1. Write all the ways to name the line.



Use the lowercase letter near the arrow: line *a*.

Use any two points in any order with a line symbol above them:
$\overleftrightarrow{NM}, \overleftrightarrow{NP}, \overleftrightarrow{MN}, \overleftrightarrow{PN}, \overleftrightarrow{MP}, \overleftrightarrow{PM}$.

2. True or false?

A segment's length can be measured.

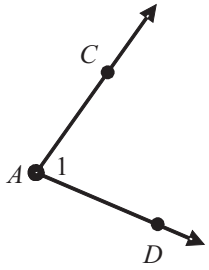
A segment is part of a line. It does not go on forever in any direction. The distance from endpoint to endpoint can be measured.

The statement is true.

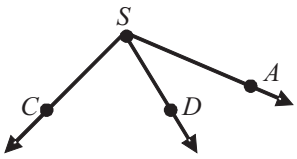
Expand Their Horizons

In this subtopic, students are introduced to angles and how to name them. An angle is formed when two distinct (or different) rays are joined at their endpoints. The point where they are joined is called the vertex. The rays are called the sides of the angle. If the two rays are facing in opposite directions, their union will look like a line. This is called a straight angle and is defined in the next lesson.

There are three ways to name an angle: by its vertex only, by three points (one on one side, the vertex, and one on the other side), or by a number located near the vertex. The angle symbol, $\angle$, precedes the letters or number used. The angle below can be named $\angle A$, $\angle CAD$, $\angle DAC$, or $\angle 1$.

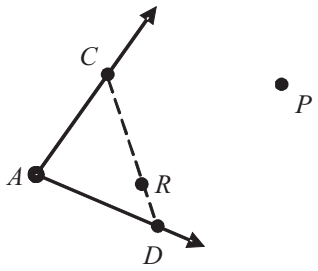


Numbers are frequently used when there are several angles in the same diagram. When two angles in a diagram have the same side, such as shown below, neither angle can be named only by its vertex because it would not be clear which angle is being referred to.



An angle divides a plane into three parts: The interior of the angle, the exterior of the angle, and the angle itself. Because a plane extends forever in all directions, the interior and exterior of an angle are the same "size." They both occupy an infinite amount of space.

One way to determine if a point is in the interior of an angle is to see if it lies on a line segment whose endpoints lie on the angle. In the figure below, point *R* is in the interior of $\angle CAD$ because *R* lies on line segment *CD*. Point *P* also lies in the interior because the rays can be extended indefinitely.

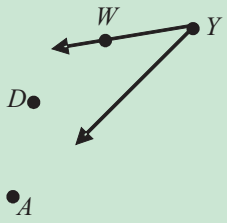


5 The vertex is the point where the two sides meet: Point *M*. The sides are the rays: $\overrightarrow{MN}$ and $\overrightarrow{MR}$.

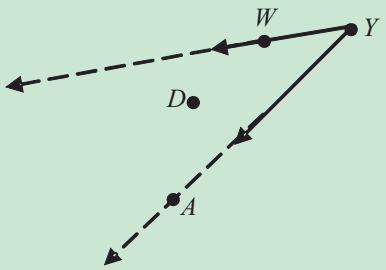
6 The points on the angle lie on either ray: Points *T*, *M*, and *B*. Points *R* and *L* are in the interior because they are not on the angle but would be on a segment whose endpoints are on the angle. The remaining points, *G* and *Q*, are in the exterior.

Additional Examples

1. Tell whether each point shown lies on the angle, in the angle's interior, or in the angle's exterior.

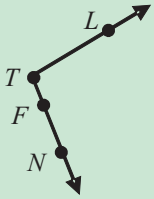


Extend the rays.



Points *W*, *Y*, and *A* are on the angle. Point *D* is in the interior. There are no points in the exterior.

2. Write all the ways to name the angle.



Name the angle by its vertex: $\angle T$.

Name the angle using three points. Be sure the vertex is the second point and one point from each side make up the other two points:

$\angle LTF$, $\angle LTN$, $\angle NTL$, $\angle FTL$.

Look Beyond

In the next lesson, students will continue to learn about angles. They will learn how angles are measured using a protractor and will learn how to estimate the measure of an angle. They will also classify angles based on their angle measurement.

Students will also explore line relationships. Lines can intersect each other to form right angles, in which case the lines are perpendicular. Lines can intersect and not be perpendicular, or lines can not intersect each other at all, in which case they are called parallel. Because directions in everyday life refer to intersections and parallel and perpendicular streets, it is critical that students understand these relationships.

Connections

The word *geometry* means *to measure the earth*. Geometry grew out of a need to measure and survey the land. The ancient Greeks invented instruments which, with geometric principles, allowed them to measure angles and distances. Modern surveyors still rely on geometry to calculate land measurements. Properties of triangles can be used to measure distances that are difficult to measure directly. Diameters and radii of circles are used when determining the curvature of a road and when determining the maximum driving speed which should be posted for that road.