Numbers and Operations

Module 7

Ratio, Proportion, and Percent

Lesson 7
Problem Solving with Percents

Teacher Notes 7.7

Objectives

- Solve real-world percent problems including percent of increase and decrease with or without technology.
- Solve real-world percent problems involving simple and compound interest with or without technology.

POWER TALK SEND

Prerequisites

Converting among fractions and mixed numbers, decimals, and percents

Multiplying whole numbers, decimals, and fractions

Finding the percent of a number

Vocabulary

Percent (4.1)
Markup (7.5)
Product (2.3)
Interest
Percent of change
Simple Interest
Principal
Compound interest

Get Started

- Divide students into two groups so that each group has an even number of students.
- Divide Group One into two subgroups: the percents and the whole numbers. Give each student a blank paper. Instruct the percents to write a percent on their paper and the whole numbers to write a whole number on their paper.
- Have students fold their papers so no one can see what is written inside of the papers. Then randomly pair up each percent with a whole number.
- Have pairs stand next to each other in front of the class with their papers unfolded and facing Group Two, who are still in their seats. Have Group Two find the value of each pair.

- Repeat with Group Two dividing into percents and whole numbers and Group One determining the value of each pair.

- In today's lesson, we will continue to use percents by applying them to real-life problems.

Subtopic 1 — Percent of Increase and Percent of Decrease

Expand Their Horizons

In this subtopic, students find a percent of change by dividing the amount of change by the original amount. This can be written as: percent of change = $\frac{\text{amount of change}}{\text{original amount}}$. If the new amount is greater than the original amount, the percent is a percent of increase. If the new amount is less than the original amount, it is a percent of decrease.

As with finding other percents, students can also use the percent proportion to find the percent of increase or decrease. The amount of change is the *part* and the original amount is the *whole*. In the lesson, 25 photos were taken last week, and 45 photos were taken this week. To find the percent of increase with the formula, students divide the amount of increase, 20, by the original amount, 25. To find the percent of increase with the percent proportion, use $\frac{20}{25} = \frac{p}{100}$. To solve, multiply 20 by four because $25 \times 4 = 100$ in the denominators: $\frac{20 \times 4}{25 \times 4} = \frac{80}{100}$. It was an 80% increase.

Common Error Alert:

Students usually have no difficulty subtracting to find the amount of change; however, they may divide by the new amount rather than by the original amount. Tell students they are finding how much it changed *from the old amount*, and that is why they should always compare to the old amount.

After finding percents of change, students find new amounts when the percent of change is given. The new amount is the sum or difference of the original amount and the amount of change. Students find the amount of change by finding a percent of a number. One way to find a percent of a number is to multiply the decimal equivalent by the number. However, show students how mental math and reasoning can sometimes be used.

For example, in the lesson, an original cost of $50 is being increased by 35%. The lesson shows how to find 35% of 50 by multiplying: $0.35 \times 50 = 17.5$. Ask students how they can find this amount using mental math. One way is to think that because 35% means 35 out of 100, and 50 is half of 100, I only need half of 35, which is 17.5.

Students may realize that increasing a number by 35% is the same as multiplying that number by 100% + 35%, or 135%. Likewise, decreasing a number by 35% is the same as multiplying by 100% – 35%, or 65%.

1 The amount of change is the difference between 600 and 240, or 360. Divide 360 by the original amount of 240. Convert the quotient 1.5 into a percent: 150%. Because the new amount is greater than the old amount, it is a percent of increase.

2 Find 20% of 300 by multiplying 0.2 by 300 or by multiplying $\frac{1}{5}$ by 300: $\frac{1}{_1\cancel{5}} \times \frac{\cancel{300}^{60}}{1} = 60$. Because it is a decrease, subtract: \$300 – \$60 = \$240. Students may also choose to multiply \$300 by 80%: $0.8 \times 300 = 240$. They may think: 80% means 80 out of 100, and three groups of 80 is 240.

Additional Examples

1. Clay caught 10 fish on Monday and eight fish on Tuesday. What was the percent of change?

Subtract to find the amount of change: 2.

Divide by the original, or first, amount:

$$\frac{2}{10} = \frac{1}{5} = 20\% .$$

It was a 20% decrease.

2. The cost of admission to a theme park used to be \$40. It has increased by 30%. What is the new cost of admission?

Find the amount of increase:

30% of 40
0.3×40
12

Add to find the new cost:
\$40 + \$12 = \$52.

Subtopic 2 — Simple Interest

Expand Their Horizons

In this subtopic, students learn why banks pay and charge interest and how to compute simple interest. At this age, many students do not have a bank account and do not understand how interest works. Discuss with students why people put their money in a bank. One reason is for safety. It is safer to keep large amounts of money in a bank than to keep it at home where it can be stolen. While a bank may also be robbed, most accounts are FDIC insured up to \$100,000, meaning even if their bank is robbed, their money up to that amount is safe, because the government will repay the bank for the stolen money.

Another reason to put money in a bank is to earn interest. Explain that banks pay interest on savings accounts (and some checking accounts) to encourage people to save at their bank. That way, the bank can use money deposited by one person to lend to another person. If the bank is charging more interest to the person borrowing the money than the bank is paying interest to the person saving the money, the bank makes a profit.

Tell students that customers can make withdrawals and deposits into savings accounts at any time. However, for the problems in this lesson, it is assumed that no withdrawals or additional deposits are being made. Depending on the different level of students, teachers may wish to discuss that certain types of savings accounts, such as CDs, have penalties for early withdrawals, which is another way banks make a profit.

To calculate simple interest, multiply the principal, times the rate, times the time. The formula is written as $I = Prt$. Remind students to convert the rate to a decimal by moving the decimal point two places to the left. Also, remind them that they are working with money, so final answers should be rounded to the hundredths place when needed.

Common Error Alert:

Students may forget that time is always written in terms of years and use six instead of $\frac{1}{2}$ if the time is written as six months. It may help if students convert the interest rate to a decimal and the time to years, if needed, before writing their equation.

3 Convert the interest rate to a decimal: 0.04 and multiply: $400(0.04)(3). The simple interest earned is $48. To find the total amount, add the interest to the principal, $400 + $48 = $448.

Additional Examples

1. **Sheila saved $500 for $2\frac{1}{2}$ years at a rate of 5.5%. Find the amount of simple interest earned.**

   Identify the principal, rate, and time, and substitute into the formula, $I = Prt$.

$$I = Prt$$
$$= (\$500)(0.055)(2.5)$$
$$= \$68.75$$

   Sheila earned $68.75 in simple interest.

2. **Kevin borrowed $300 for four months at a rate of 10%. Find the amount of simple interest he must pay. Find the total amount he must pay back.**

   Four months is one-third of a year.

$$I = Prt$$
$$= (\$300)(0.1)\left(\frac{1}{3}\right)$$
$$= \$10$$

   Kevin must pay $10 in interest. The total amount he must pay back is the sum of the principal and interest:
   $100 + $10 = $110.

Expand Their Horizons

In this subtopic, students compute compound interest by calculating simple interest with the new principal for each compounding period. Explain that most banks use compound interest rather than simple interest and that the interest may be compounded as often as daily.

To find compound interest, students use the simple interest formula as many times as the interest is compounded. If interest is compounded annually for two years, the interest will be compounded twice. If interest is compounded semiannually for two years, the interest will be compounded four times, that is, every six months for two years.

When substituting into the formula $I = Prt$, students must adjust the time for how often the interest is being compounded. If the interest is compounded annually, the time is one. If the interest is compounded semiannually, the time is written as $\frac{1}{2}$ because six months, or half a year, has passed. If the interest is compounded monthly, the time is $\frac{1}{12}$ of a year, and so on.

At the end of each compounding period, students should round the interest to the nearest cent. The principal for the next period is that amount plus the previous principal. When the interest is compounded several times, it might help students to use a table such as the following to organize their work. Show how the last column in one row is the first column in the next.

Month	Principal	*Prt*	Interest	New Amount
1	\$800	$\$800(0.08)\left(\frac{1}{12}\right)$	\$5.33	\$805.33
2	\$805.33	$\$805.33(0.08)\left(\frac{1}{12}\right)$	\$5.37	\$810.70
3	\$810.70	$\$810.70(0.08)\left(\frac{1}{12}\right)$	\$5.40	\$816.10

Show how the total amount of interest earned can be found either by adding the interest amounts in the second to last column or by subtracting the original principal by the new amount in the last row. Point out also how the amount of interest increases from row to row. With simple interest, these amounts remain the same. That is why, given the same principal, rate, and time, compound interest makes more money than simple interest.

Discuss that interest rates vary from bank to bank, and if looking for a bank in which to save money, interest rates are one factor to consider. Other factors include the bank's distance from home, service fees, online banking capabilities, and minimum deposit and balance requirements.

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The interest is compounded annually, so the time is one year. The interest for the first year is ($5,000)(0.06)(1), or $300. This makes the total amount for the first year $5,300. Use this as the principal for the second year: ($5,300)(0.06)(1) = $318. This makes the total amount for the second year $5,618.

Additional Examples

1. Andre saved $10,000 at an interest rate of 5% compounded quarterly. Find the total amount at the end of one year.

Use $t = \frac{1}{4}$ in each calculation.

1st Q: ($10,000)(0.05) $\left(\frac{1}{4}\right)$ = $125

2nd Q: ($10,125)(0.05) $\left(\frac{1}{4}\right)$ = $126.56

3rd Q: ($10,251.56)(0.05) $\left(\frac{1}{4}\right)$ = $128.14

4th Q: ($10,379.70)(0.05) $\left(\frac{1}{4}\right)$ = $129.75

Total amount:
$10,379.70 + $129.75 = $10,509.45

2. Shannon borrowed $700 at an interest rate of 3.5% compounded semiannually. Find the total amount of interest she will pay at the end of one year.

Use $t = \frac{1}{2}$ in each calculation.

1st half: ($700)(0.035) $\left(\frac{1}{2}\right)$ = $12.25

2nd half: ($712.25)(0.035) $\left(\frac{1}{2}\right)$ = $12.46

She will pay $12.25 + $12.46, or $24.71 in interest.

Look Beyond

In more advanced math classes, students will use a formula to find compound interest. The interest rate is divided by the number of times the interest is compounded per year. This is added to 100%, or 1. This sum is raised to the number of compounding periods in total, which is the number of times per year times the number of years. Last, this value is multiplied by the original principal. This gives the new amount after a given number of years. The formula looks like this: $A = P\left(1 + \frac{r}{n}\right)^{tn}$ where n is the number of times the interest is compounded per year and t is the number of years.

Connections

The average person will both save and borrow money in their lifetime. Money can be borrowed as a personal loan from a bank, as a mortgage for a house, or from banks in the form of credit cards. All of these types of loans are offered with different interest rates and terms. Money can be saved in the form of a regular savings account, either from a “brick and mortar” bank or a bank which operates online to reduce costs and increase interest rates. Money can also be saved in the form of certificates of deposits, which offer higher rates but penalties for early withdrawal, and individual retirement accounts.