Numbers and Operations
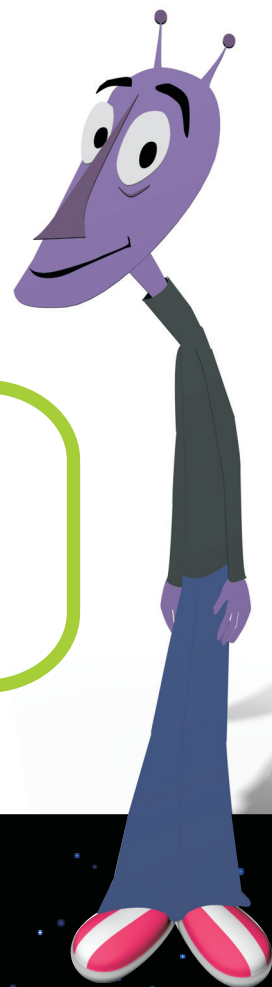
★ Module 7 ★

Ratio, Proportion, and Percent

Lesson 6
Using Percent Equations
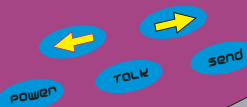
Objectives

♦ Find a percent of a given number.

♦ Find what percent one number is of another number.

♦ Find a number when a percent of it is given.

♦ Use equations to solve percent problems.

POWER   TALK   SEND

Prerequisites

Solving proportions

Writing percents as decimals and decimals as percents

Multiplying and dividing whole numbers and decimals

Vocabulary

Percent proportion (7.5)
Percent (4.1)
Product (2.3)
Proportion (7.2)
Numerator (4.1)
Denominator (4.1)
Variable

Get Started

- Split students into two groups and have them line up on opposite sides of the room facing the board.

- Game play starts with Group One. Write a fraction with a value relatively close to zero, such as $\frac{1}{10}$. Have the first student in line come to the board and write a decimal with a value greater than $\frac{1}{10}$. If it is correct, have that student go to the end of the line and the next student come to the board and write a percent greater than the decimal that was just written. If that is correct, have that student go to the end of the line and the next student come up and write a fraction greater than that percent and so on. Each student is to write a value greater than the previous value so that the types of numbers alternate in the pattern: fraction, decimal, percent.

7.6A

- Game play for Group One stops when someone makes a mistake. Count the number of correct answers. That is Group One's total points. Then, have Group Two play. Start them with a different fraction than Group One, such as $\frac{1}{12}$ or $\frac{1}{8}$.

- After both groups have played "greater than," start each group with a fraction relatively close to one and have each student write a value *less than* the previous number.

- Tell students that today's lesson will require that they be fluent in converting between fractions, decimals, and percents.

Subtopic 1 — Find the Percent of a Number

Expand Their Horizons

In this subtopic, students review the percent proportion and then, use it to solve problems to find a percent of a number, such as in the question: *What is 30% of 50?* To solve using the percent proportion, identify the three components: part, whole, and percent. In this case, the part is the unknown, the percent is 30, and the whole is 50. Therefore, the percent proportion $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ becomes $\frac{?}{50} = \frac{30}{100}$. This proportion can be read as: *What number out of 50 equals 30 out of 100* or *what number out of 50 equals 30%*. Remind students, that in the percent proportion, it does not matter which ratio is written first; therefore, the equation could also be written as $\frac{30}{100} = \frac{?}{50}$, which would be read as *30 out of 100 equals what number out of 50*, or *30% equals what number out of 50*.

Students are also reminded that because *of* means multiply, they can find the percent of a number by changing the percent to a decimal and by multiplying. In the above example, this would be 0.3×50, or 15. Students who wish to directly translate the entire sentence to a mathematical sentence would write $? = 0.3 \times 50$.

Encourage students to reason and to use mental math when the numbers are easy to work with. For instance, in the example mentioned above, 10% of 50 is five. Then, 30% is $3 \times 10\%$, so 30% is 3×5, or 15.

1 The percent is 30, the whole is 250, and the part is unknown. Substitute into the percent proportion to get $\frac{?}{250} = \frac{30}{100}$. Solve the proportion. Because the ratios are equivalent in a proportion, one way to solve is to multiply the numerator by 2.5 because $100 \times 2.5 = 250$ in the denominators. The proportion can also be solved by using cross products: $? \times 100 = 250 \times 30$. Multiply on the right: $? \times 100 = 7,500$. Find the missing factor by dividing by 100: $7,500 \div 100 = 75$ of the seats are filled.

2 To find 300% of 12, find 300% × 12, or 3 × 12. If choosing to use the percent proportion, the percent is 300, the whole is 12, and the part is the unknown: $\frac{?}{12} = \frac{300}{100}$. Using cross products: ? × 100 = 12 × 300. This simplifies to ? × 100 = 3,600. The unknown is $36.

Additional Examples

1. **Sarah bought a ring for 65% of what it was worth. The ring was worth $800. How much did Sarah pay for the ring?**

    The question can be worded as: what is 65% of $800?

    $$65\% \text{ of } 800$$
    $$0.65 \times 800$$
    $$520$$

    Using the percent proportion:

    $$\frac{?}{800} = \frac{65}{100}$$
    $$\frac{520}{800} = \frac{65 \times 8}{100 \times 8}$$

    Sarah paid $520 for the ring.

2. **The value of Nate's comic book is 175% of its original value of $2.50. Find the value of Nate's comic book now.**

    What is 175% of $2.50?

    $$175\% \text{ of } 2.5$$
    $$1.75 \times 2.5$$
    $$4.375$$

    Using the percent proportion:

    $$\frac{?}{2.5} = \frac{175}{100}$$
    $$\frac{4.375}{2.5} = \frac{175 \div 40}{100 \div 40}$$

    Rounding to the nearest hundredth for money, the value is $4.38.

Subtopic 2 ▼ Find What Percent One Number Is of Another Number

Expand Their Horizons

In this subtopic, students are given the part and the whole and must find the percent. A typical question would be: *What percent of 80 is 20?* Point out that this is the same as asking: *Twenty is what percent of 80?* Regardless of how it is worded, the part is 20 and the whole is 80. In both wordings, the whole follows the word *of* and the part follows or precedes the word *is*. Because of this, some teachers write the percent proportion as $\frac{is}{of} = \frac{percent}{100}$; therefore, some students may have seen and may want to use this "is over of" method. Encourage these students to write the proportion using *part* and *whole*. Not only will it help them solve word problems now, but using correct terminology will help them in future math classes.

Students are also introduced to the term *variable* in this subtopic. A variable is a letter or symbol used to represent a number. Since a question mark is a symbol, they have been using variables all along. Tell them that letters are the most common type of variable and used extensively in high school math. Any letter can be used. In higher math, Greek letters are used too.

7.6C

Some students may automatically write the lesser number as the part and the greater number as the whole. Remind students that the part is only less than the whole when the percent is less than 100, but percents may be greater than 100 as well. Show students the difference with these examples.

What percent of 80 is 20?	What percent of 20 is 80?
Part: 20	Part: 80
Whole: 80	Whole: 20

$$\frac{20}{80} = \frac{n}{100}$$

$$20 \times 100 = 80 \times n$$
$$2{,}000 = 80 \times n$$
$$2{,}000 \div 80 = n$$
$$25 = n$$

$$\frac{80}{20} = \frac{n}{100}$$

$$80 \times 100 = 20 \times n$$
$$8{,}000 = 20 \times n$$
$$8{,}000 \div 20 = n$$
$$400 = n$$

In the first, the part is less than the whole, and the percent is 25%. In the second, the part is greater than the whole, and the percent is 400%.

The lesson also shows how to write an equation from the question by replacing the unknown with a variable, *of* with times, and *is* with an equal sign. For example, *what percent of 80 is 20* is written as $n \times 80 = 20$. To solve, divide 20 by 80 and convert the decimal to a percent. Some students may notice that to find the percent, they simply have to divide the part by the whole, and they convert the decimal to a percent.

No matter which way students choose to solve their problems, they should always check the reasonableness of their answers by estimating, and/or check the answer directly by finding the percent of the number. For instance, to check if 25% is the answer to *what percent of 80 is 20*, they should find 25% of 80 and should see if the answer is indeed 20: $\frac{1}{4} \times 80 = 20$.

3 Since the question asks for the percent spent, first find the amount spent. This is the part. The whole is the original amount, $140. The percent proportion is $\frac{28}{140} = \frac{n}{100}$. Solve by using equivalent fractions and by dividing 28 by 1.4, or by using cross products: $28 \times 100 = 140 \times n$, which becomes $2{,}800 = 140 \times n$. Divide to find the missing factor of 20%.

Students choosing to write an equation for *what percent of 140 is 28* should write $n \times 140 = 28$, divide 28 by 140, and convert the decimal to a percent.

4 In this problem, the part is greater than the whole. The part is 75 and the whole is 50. The percent proportion is $\frac{75}{50} = \frac{z}{100}$, and the equation is $z \times 50 = 75$. While solving this proportion, students should notice that $50 \times 2 = 100$, so they would multiply 75 by two to get 150. If solving the equation, they would divide 75 by 50 to get 1.5 and would move the decimal point two places to the right: 150%.

7.6D

Additional Examples

1. Ten is what percent of 80?

The part is 10 and the whole is 80.

$$\frac{10}{80} = \frac{p}{100}$$
$$1{,}000 = 80 \times p$$
$$12.5 = p$$

Another way is to write an equation.

Ten is what percent of 80?
$$10 = p \times 80$$
$$10 \div 80 = p$$
$$0.125 = p$$
$$12.5\% = p$$

Ten is 12.5% of 80.

2. John has 12 pencils in his backpack, but only four of them are sharpened. What percent of John's pencils are sharpened?

The part is four and the whole is 12.

$$\frac{4}{12} = \frac{p}{100}$$
$$400 = 12 \times p$$
$$33.\overline{3} = p$$
$$33\frac{1}{3}\% = p$$

Another way is to write and solve an equation for *what percent of 12 is four*:

$$p \times 12 = 4$$
$$p = 4 \div 12$$
$$p = 0.\overline{3}$$
$$p = 33\frac{1}{3}\%$$

John has $33\frac{1}{3}\%$ of his pencils sharpened.

Subtopic 3 — Find a Number When a Percent of It Is Given

Expand Their Horizons

In this subtopic, students are given the part and the percent and should find the whole. For example, *50% of what number is 80?* In this example, 50 is the percent and 80 is the part. The proportion would be $\frac{80}{w} = \frac{50}{100}$. It may help students to understand the question if the percent were read as a fraction. In the problem above, the question becomes: *One-half of what number is 80?* This may make it more apparent that the answer is 160.

Encourage students to estimate answers. Just by looking at the percent, one can tell if the whole will be greater or less than the part. If the percent is less than 100, the whole will be greater than the part, as in the example above.

In the lesson, students are shown how to answer: *Twenty-five percent of what number is 150?* Note that the question could have also been worded as: *One hundred fifty is 25% of what number?* The percent proportion is $\frac{150}{w} = \frac{25}{100}$ which is solved by either multiplying 100 by six because $25 \times 6 = 150$, or by using cross-products: $15{,}000 = 25 \times w$.

7.6E

Again, students can choose to write and to solve an equation by translating the sentence into mathematics: 25% × w = 150, which is 0.25 × w = 150.

5 The part is 30 and the percent is 15. Solve the proportion $\frac{30}{m} = \frac{15}{100}$. Because 15 × 2 = 30, multiply 100 by two, to find that m = 200. The question can also be translated into 0.15 × m = 30.

6 Because the percent is greater than 100%, the whole will be less than the part. The question can be translated into 200% × n = 700, which is 2 × n = 700. The missing factor is 350. To solve with a proportion, write $\frac{700}{n} = \frac{200}{100}$. Either multiply 100 by 3.5 because 200 × 3.5 = 700 or use the cross products: 70,000 = 200 × n.

Additional Examples

1. **An athlete's lunch contain 840 calories. This is 28% of the number of calories his trainers want him to consume in one day. How many calories do the trainers want the athlete to consume in one day?**

   Answer the question:
   28% of what number is 840?

   This translates into the equation:
   0.28 × w = 840.

   Solve by dividing 840 by 0.28: 3,000.

   The trainers want the athlete to consume 3,000 calories per day.

2. **1,000% of what number is 20?**

   The part is 20; the percent is 1,000.

   $$\frac{20}{w} = \frac{1,000}{100}$$
   $$\frac{20}{2} = \frac{1,000 \div 50}{100 \div 50}$$

   1,000% of two is 20.

Look Beyond

Students will continue to solve percent problems to find percent of increase, percent of decrease, and new amounts resulting from these changes in the next lesson. They will be able to find their percents by either using the percent proportion or by writing an equation. Students will also work with percents to solve problems involving simple and compound interest.

7.6F

Connections

Nutritionists work with percents when creating diets for their clients. Based on the clients needs, a nutritionist will determine what percent of their client's diet should be comprised of carbohydrates, protein, and fat. They will also consider how many calories their client should consume, on average, per day and perhaps will determine what percent should be eaten for breakfast, lunch, and dinner. Using percents, the nutritionist can be sure to plan a well-rounded diet for the client, which includes all the necessary nutrients in the required proportions.

7.6G