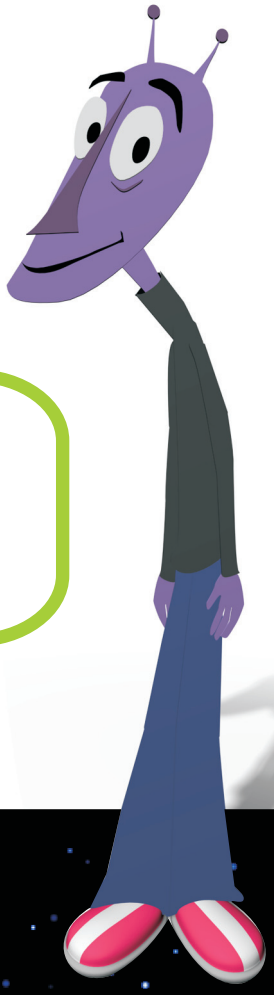
Numbers and Operations

Module 7

Ratio, Proportion, and Percent

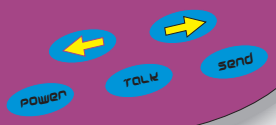


Lesson 5
Percent Proportions

Teacher Notes 7.5

Objectives

- Use the percent proportion to write fractions as percents.
- Determine the percent of a number and solve related problems in real-world situations (e.g. gratuities, sales tax, discounts, and mark up).
- Use percents to estimate.



Prerequisites

Writing and solving proportions

Finding the percent of a number

Adding, subtracting, multiplying, and dividing whole numbers and decimals

Vocabulary

Percent proportion
Percent (4.1)
Proportion (7.2)
Cross products (7.2)
Ratio (4.1)
Numerator (4.1)
Denominator (4.1)
Compatible numbers (1.5)
Markup
Discount

Get Started

- Write $\frac{2}{6} = \frac{3}{12}$ on the board. Ask students if the statement is correct. **No, the cross products are not equal.**
- Erase the first term: $\frac{\ }{6} = \frac{3}{12}$. Ask students what number should be the first term to make the statement true and why. **1.5; Possible answer: In the denominators, 12 divided by two is six, so divide three by two.**
- Erase the second term: $\frac{2}{\ } = \frac{3}{12}$. Ask students what number should be the second term to make the statement true and why. **8; Possible answers: In the numerators, three divided by 1.5 is two, so divide 12 by 1.5 or think of a denominator that makes the fraction equivalent to $\frac{1}{4}$ because $\frac{3}{12}$ simplifies to $\frac{1}{4}$.**

- Erase the third term: $\frac{2}{6} = \frac{}{12}$. Ask students what number should be the third term to make the statement true and why. **4; Possible answers: In the denominators, six times two is 12, so multiply two by two or think of a numerator that makes the fraction equivalent to $\frac{1}{3}$ because $\frac{2}{6}$ simplifies to $\frac{1}{3}$.**
- Erase the fourth term: $\frac{2}{6} = \frac{3}{}$. Ask students what number should be the fourth term to make the statement true and why. **9; Possible answers: In the numerators, two times 1.5 is three, so multiply six by 1.5 or think of a denominator that makes the fraction equivalent to $\frac{1}{3}$ because $\frac{2}{6}$ simplifies to $\frac{1}{3}$.**

Subtopic 1 — Use the Percent Proportion to Write Fractions as Percents

Expand Their Horizons

In this subtopic, students are introduced to the percent proportion. In a percent proportion, one of the ratios has a denominator of 100. Since a percent is a number out of 100, the numerator of that ratio is a percent. The other ratio is equivalent to that percent.

The percent proportion is shown as $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ although the ratios may be switched: $\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$. Students are shown that to convert a fraction to a percent, they can substitute the numerator of the fraction for the *part* and the denominator of the fraction for the *whole*. Then, solve the proportion for the percent. For example, to write $\frac{3}{4}$ as a percent, the percent proportion becomes $\frac{3}{4} = \frac{?}{100}$. To solve, either use cross products or form equivalent ratios by multiplying by 25: $\frac{3\times25}{4\times25} = \frac{75}{100}$. The fraction $\frac{3}{4}$ is equivalent to 75 out of 100, or simply 75%.

1 Because five out of 250 bulbs were defective, five is the part and 250 is the whole. Solve the proportion by writing equivalent fractions. In the denominators, 250 ÷ 2.5 = 100, so divide five by 2.5 in the numerators. The equivalent percent is the new numerator: 2%.

2 To find the percent more games, first, find how many more games they won this year: four. Because this year is being compared to last year, the number of games won last year is the whole. The percent proportion is $\frac{4}{8} = \frac{?}{100}$. Students who do not see that $\frac{4}{8}$ simplifies to $\frac{1}{2}$ may multiply four in the numerator by 12.5 to form equivalent fractions. Encourage students to simplify the fraction when it makes computations easier and/or simplifies to a benchmark fraction: $\frac{1}{2} = \frac{?}{100}$. The percent is 50%.

Additional Examples

1. Use the percent proportion to write $\frac{3}{15}$ as a percent.

Set up the percent proportion: $\frac{3}{15} = \frac{?}{100}$.

Simplify the first ratio: $\frac{1}{5} = \frac{?}{100}$.

Form equivalent ratios: $\frac{1 \times 20}{5 \times 20} = \frac{20}{100}$.

The fraction $\frac{3}{15}$ is equivalent to 20%.

2. Out of 120 bananas shipped to a market, six were rotten. What percent of the shipment was rotten?

Set up the percent proportion: $\frac{6}{120} = \frac{?}{100}$.

Form equivalent ratios: $\frac{6 \div 1.2}{120 \div 1.2} = \frac{5}{100}$.

Five percent of the shipment was rotten.

Subtopic 2

Estimate a Percent of a Number

Expand Their Horizons

In this subtopic, students learn several methods to estimate a percent of a number. One common method involves rounding the percent to the nearest 10. For instance, to estimate 38% of 25, first round 38% to 40%. Find 10% of the number by moving the decimal point in the number one place to the left: 2.5. Then, multiply. Since 10% of 25 is 2.5, 40% of 25 is 4 × 2.5, or 10.

In the lesson, students see that to estimate 29% of 320, they can find 30% of 320, which is 3 × (10% of 320) or 3 × 32, for an estimate of 96. Alternatively, students can round the percent to a benchmark percent and write the equivalent fraction. In this case, 29% rounds down to 25% and is written as $\frac{1}{4}$. Then multiply: $\frac{1}{4} \times 320 = 80$. Both estimates are reasonable. The first method rounded 29% up to 30%, and the second method rounded 29% down to 25%. The actual answer is somewhere between the two estimates but closer to the estimate using 30% because 29% is closer to 30% than 25%.

Students can also use mental math and reasoning. Another way to find 30% of 320 is to think: 30 for every 100 makes 90. The remaining 20 out of 100 would be $\frac{1}{5}$ of 30, or six, for an estimate of 96.

For small percents, students can also find 1% of the number by moving the decimal point two places to the left. To estimate 6% of 188, they would find 1% of 200, which is two. They then would multiply by six: 6 × 2 = 12.

Tell students they have the option of rounding the percent, the number, or both. When deciding what number to round to, they should use compatible numbers. This may mean rounding up or down.

3

To estimate 42% of $880, round the percent to 40% and the dollar amount to $900 because 4 × 9 is a basic multiplication fact. Another option is to write 40% as $\frac{2}{5}$ and multiply: $\frac{2}{5} \times \$900 = \$360$.

Additional Examples

1. Estimate 82% of 1,530.

Round 82% to 80% and 1,530 to 1,500.

Ten percent of 1,500 is 150, so 80% is 8 × 150, or 1,200.

Or, write 80% as $\frac{4}{5}$ and multiply.

$$\frac{4}{\cancel{5}_{1}} \times \cancel{1{,}500}^{300} = 1{,}200$$

2. Estimate 4.5% of 14.

Estimate 4.5% of 14 by finding 5% of 14.

Find 10% of 14: 1.4.

Five percent is half of 10%, so divide 1.4 by two: 1.4 ÷ 2 = 0.7.

Subtopic 3 — Determine the Percent of a Number and Solve Problems

Expand Their Horizons

In this subtopic, students find the percent of a number to determine sales tax, tips, markups, and discounts. In the lesson, students see how to use the percent proportion to find 6.25% of $200. In this case, the percent is known, and the unknown is the part. Another way students can find the sales tax is to multiply 0.0625 × $200.

Next, students learn that the markup on an item is an amount added to the cost to produce the item, which includes cost of materials and labor. Another type of markup is the amount a store adds to what they paid for the item. The percent of markup is sometimes called the markup rate. Sometimes it is greater than 100%.

Common Error Alert:

Students may find the amount of markup and may forget to add that amount to the cost, especially when the markup rate is greater than 100% and when the amount of markup is greater than the cost to produce the item. Remind students that markup is not the selling price; it is an amount used to determine the selling price. If students continually make this mistake, have them write the equation, *cost + markup = selling price*, in their work space.

A discount is a reduction in the cost of an item. It can also be called a markdown. Like a markup, a discount is not the selling price but is used to determine the selling price. After determining the discount, subtract the discount from the original price to determine the new selling price.

Teachers may wish to mention to students that most stores will mark up items at a rate which is several hundred percent of what they paid for it. This way, even when they reduce the item by great amounts, such as 80% off, they still make a profit.

4

Find 15% of the total bill which is $12. Although students can multiply 0.15 by $12, remind them that in a restaurant, without pencil and paper, mental math is often the best way to determine a tip. To find 10% of $12, move the decimal point one place to the left: $1.20. To find the additional 5%, divide $1.20 by two: $1.20 ÷ 2 = $0.60. Add the two amounts: $1.20 + $0.60 = $1.80.

Additional Examples

1. The markup on a dog leash is 200% of the cost to produce it. Find the selling price if the cost to produce it is $1.50.

Find the amount of markup: 200% of $1.50.

$$200\% \times \$1.50$$
$$2 \times \$1.50$$
$$\$3$$

Add to find the selling price.

$$\$1.50 + \$3 = \$4.50$$

The selling price of the leash is $4.50.

2. James bought a new TV for $630. The sales tax was 5%. How much, in all, did James pay for the TV?

Write the percent proportion, with five as the percent and 630 as the whole.

$$\frac{?}{630} = \frac{5}{100}$$

Solve the proportion.

$$\frac{31.5}{630} = \frac{5 \times 6.3}{100 \times 6.3}$$

The tax is $31.50. Add the tax to the cost of the TV to find the total amount James paid: $630 + $31.50 = $661.50.

Look Beyond

In the next lesson, students will continue to use the percent proportion. They will solve for the *part* to find the percent of a given number. They will solve for the *percent* to find what percent one number is of another, and they will solve for the *whole* to find a number when a percent of it is given. These are the three types of percent problems. Although they can all be solved by using the percent proportion, students will see that it is sometimes easier to solve them by other means, such as writing an equation that is not a proportion. The choice on how to solve the proportion will depend on the compatibility of the numbers given.

Connections

Food servers rely on the generosity of customers to make a decent living. Sometimes for parties greater than six or eight people, a certain percent of gratuity is automatically applied to the bill. This can also happen to smaller parties if they are dining at an unusual hour, such as between midnight and 6 AM. If a customer receives excellent service, they may choose to add to the amount that was automatically applied. Customers should also be aware that 100% of their tip does not always go directly to their server, especially when dining in finer restaurants. Tips are often broken down, percent wise, to others who helped make the meal enjoyable, such as bus people and hostesses.