Numbers and Operations

Module 7

Ratio, Proportion, and Percent

Lesson 3
Decimal and Percent Equivalents

Teacher Notes 7.3

Objectives

- Convert proper fractions to decimals and percents.
- Convert mixed numbers and improper fractions to decimals and percents.

power talk send

Prerequisites

Dividing decimals

Writing fractions as decimals

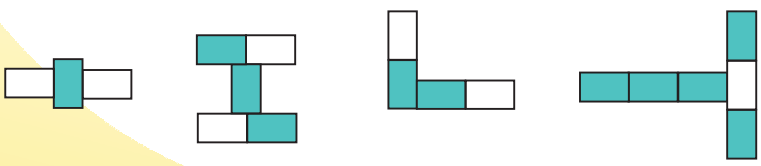
Writing equivalent fractions

Vocabulary

Proper fraction (4.1)
Numerator (4.1)
Denominator (4.1)
Mixed number (4.2)
Quotient (2.4)
Finite
Terminating decimal (5.5)
Repeating decimal (5.5)

Get Started

- Sketch each of the following shapes on the board. Shade the same amount as shown here.

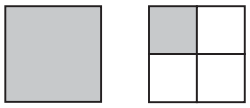


- Tell students to write down what fraction of each shape is shaded.
- Write the following percents on the board: 50%, 60%, $83\frac{1}{3}\%$, $33\frac{1}{3}\%$. Tell the students to write down what percent of each shape is shaded.

- Have students share their fraction and percent responses for each picture and their reasoning for their responses.

$$\frac{1}{3} = 33\frac{1}{3}\%,\ \frac{3}{5} = 60\%,\ \frac{1}{2} = 50\%,\ \frac{5}{6} = 83\frac{1}{3}\%$$

- Sketch the following picture on the board. Ask the students to write down what fraction and what percent of the picture is shaded. Have students share their responses.



$\frac{5}{4}$ or $1\frac{1}{4}$ and 125%

- Tell students that today's lesson will be on finding the decimal and percent equivalents for fractions and mixed numerals.

Subtopic 1 — Decimal and Percent Equivalents for Proper Fractions

Expand Their Horizons

In this subtopic, students review the characteristics of proper fractions and learn how to convert a proper fraction to a decimal and then to a percent. Any fraction can be converted to a decimal by first dividing the numerator by the denominator. For a proper fraction, this will require adding zeros as placeholders for the decimal division. Once a decimal is found, a percent can be found by multiplying the decimal by 100. The quick way to do this is to simply move the decimal point two places to the right.

An alternative to this approach is to write an equivalent fraction with a denominator of 100. Then, the numerator is the percent. This is best for problems whose denominator is a factor or multiple of 100. Remind students they can form equivalent fractions either by multiplying or by dividing both parts of the fraction by the same number.

Many students prefer to learn one way to solve a problem and resist learning other methods once they have mastered a way that works. The decision to let them use one way all the time or to encourage them to learn several methods will depend upon the students' individual learning needs and style. Stress that although division will always work, there are times when finding an equivalent fraction will be much quicker.

1

Divide one by 20. Because 20 does not divide evenly into one, add zeros as placeholders. Move the decimal point in the quotient two places to the right to form the percent.

To use the method finding an equivalent fraction with a denominator of 100, think, "Twenty times what number is 100?" The answer is five. Multiply the numerator by five: $1 \times 5 = 5$. The percent is 5%. Move the decimal two places left to find the decimal: 0.05.

2 Divide seven by 50 and move the decimal point two places to the right. Another way is to form an equivalent fraction by multiplying both the numerator and denominator by two. The numerator 14 is the percent equivalent, so 0.14 is the decimal equivalent.

Additional Examples

1. Find the decimal and percent equivalents.

$$\frac{7}{8}$$

Divide seven by eight.

$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ -64 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Move the decimal two places right for the percent equivalent: 87.5%.

2. Find the decimal and percent equivalents.

$$\frac{3}{200}$$

Form an equivalent fraction by dividing the numerator and denominator by two.

$$\frac{3 \div 2}{200 \div 2} = \frac{1.5}{100}$$

The percent equivalent is 1.5%, and the decimal equivalent is 0.015.

Subtopic 2 Decimal and Percent Equivalents for Repeating Decimals

Expand Their Horizons

In this subtopic, students work with repeating decimals. The concept of a repeating decimal is not a new one to students; they were introduced to repeating decimals when they divided decimals. What is new is how to handle the repeating part when the decimal is converted to a percent.

Begin as with terminating decimals: move the decimal point two places to the right. Then, convert the remaining decimal portion to a fraction. For example, in $\frac{2}{11}$ the repeating decimal is $0.\overline{18}$ in bar notation. Write the repeating block a few times: 0.181818... and then, move the decimal two places to the right: 18.1818...%. Note that the repeating part in this example is the same as for the original decimal, which is equivalent to $\frac{2}{11}$. Therefore, the percent can be written as $18\frac{2}{11}\%$.

Common Error Alert:

Although there will be cases where the fractional part of the percent is the same as the original fraction, there will also be cases where this is not true. For example, in $\frac{7}{30}$, the decimal equivalent is $0.2\overline{3}$. When the pattern is written out a few times, it looks like 0.233333... . The percent becomes 23.3333...%, or $23\frac{1}{3}$%. The fractional part is not $\frac{7}{30}$ because the first digit, two, is not part of the repeating pattern. The two shows up only in the whole number part of the percent.

Remind students that "repeating three" is equivalent to $\frac{1}{3}$, and that "repeating six" is equivalent to $\frac{2}{3}$.

3 Divide two by three. The decimal equivalent is $0.\overline{6}$, which looks like 0.666666.... Move the decimal point two places to the right: 66.6666... %. The decimal part of the percent is equivalent to the original fraction. The percent equivalent is written as $66\frac{2}{3}$%.

4 The quotient of the numerator divided by the denominator is $0.\overline{63}$; both the six and the three repeat. The percent equivalent is found by moving the decimal point two places to the right: 63.6363... %, which is the same as $63\frac{7}{11}$%.

Additional Examples

1. Find the decimal and percent equivalents.

$$\frac{11}{12}$$

First divide.

$$12\overline{)11.000}\quad 0.91\overline{6}$$

$$\begin{array}{r} -108 \\ \hline 20 \\ -12 \\ \hline 80 \\ -72 \\ \hline 8 \end{array}$$

The decimal equivalent can be written as 0.9166666.... Move the decimal point two places to the right: 91.6666.... This is $91\frac{2}{3}$%.

2. Find the decimal and percent equivalents.

$$\frac{8}{15}$$

First divide.

$$15\overline{)8.00}\quad 0.5\overline{3}$$

$$\begin{array}{r} -75 \\ \hline 50 \\ -45 \\ \hline 5 \end{array}$$

The decimal equivalent can be written as 0.533333.... Move the decimal point two places to the right: 53.3333.... This is $53\frac{1}{3}$%.

Expand Their Horizons

In this subtopic, students convert mixed numbers to decimals and percents. To convert a mixed number to a decimal, find the decimal for the fractional part of the mixed number and then, add the whole number. To convert to a percent, move the decimal point two places to the right as before.

When finding the decimal for the fractional part, students can use either of the procedures shown earlier in the lesson. Point out that the fractional part may be either a terminating or repeating decimal.

5 The fraction $\frac{3}{4}$ is equivalent to 0.75. Add the whole number five. The sum is 5.75, which is equivalent to 575%.

Additional Examples

1. Find the decimal and percent equivalents.

$$7\frac{3}{25}$$

Find the decimal equivalent for $\frac{3}{25}$ first.

$$\frac{3 \times 4}{25 \times 4} = \frac{12}{100}$$

Then, add three.

$$3 + 0.12 = 3.12.$$

The decimal equivalent is 3.12.
The percent equivalent is 312%.

2. Find the decimal and percent equivalents.

$$15\frac{1}{9}$$

Convert $\frac{1}{9}$ to a decimal.

$$\frac{1}{9} = 0.\overline{1}$$

Add 15.

$$15 + 0.\overline{1} = 15.\overline{1}$$

The decimal equivalent can be written as 15.111111…. Move the decimal point two places to the right: 1,511.1111... %, or $1{,}511\frac{1}{9}\%$.

Look Beyond

Students will continue to work with fractions, decimals, and percents in this module, including when they find percent increase and percent decrease. Fluency in converting from any of these forms to the other will help with estimating answers, finding answers, and checking their work. Because so many financial concerns involve percents, it is critical, when students face the world on their own, they leave school with a clear understanding of percents.

Connections

Teachers change fractions to percents when they grade papers. If a student scores 18 out of 25 questions correct, the fraction $\frac{18}{25}$ can be written as 72%. A teacher who knows that he or she can simply multiply 18×4 to find the percent will save precious time on paperwork. Percents are a large part of everyday life, not just in quizzing and testing, because it makes for easy comparisons. For example, are 18 out of 25 better or worse than 40 out of 50? Compare the percents to find $72\% < 80\%$.