Numbers and Operations

Module 7

Ratio, Proportion, and Percent

Lesson 2
Finding Percents

Teacher Notes 7.2

Objectives

- Relate with or without models and pictures, concepts of ratios, proportion, and percent, including percents less than 1 and greater than 100.
- Demonstrate conceptual understanding to find a specific percent of a number, using models, real life examples, or explanations.

Prerequisites

Writing fractions as decimals, decimals as percents, and percents as fractions and decimals

Converting improper fractions to mixed numbers

Multiplying fractions and decimals

Simplifying fractions

Vocabulary

Ratio (4.1)
Percent (4.1)
Improper fraction (4.1)
Proportion
Means
Extremes
Cross products

Get Started

- Tell students it takes four pounds of potatoes to make one bag of potato chips. Ask students for a ratio of potatoes to potato chips. **four to one**
- Ask how many pounds of potatoes will be needed to make two pounds of chips. **eight** Show that this ratio can be written as eight to two.
- Tell students to compare the two ratios. Ask what they see. **Possible answers: Each term in the second ratio is twice each term in the first ratio. They are equivalent fractions when the ratios are written as fractions.**
- Write each ratio as a fraction. Show they are equivalent because $\frac{8 \div 2}{2 \div 2} = \frac{4}{1}$.
- Ask students to write another equivalent ratio for $\frac{4}{1}$. **Possible answer: $\frac{12}{3}$**.

- Have students explain what this ratio means in terms of potatoes and potato chips. **Possible answer: It takes 12 pounds of potatoes to make three pounds of chips.**

Subtopic 1 — Percent and Ratio

Expand Their Horizons

In this subtopic, the meaning of ratio and percent are reviewed, and students learn how to write ratios as percents. If the ratio is not already written in fractional form, then write the ratio as a fraction. Next, convert the fraction to a decimal by dividing. Last, convert the decimal to a percent by moving the decimal point two places to the right.

Encourage students to estimate to check their answers. For instance, if the numerator is greater than the denominator, the percent will be greater than 100%. If the first term of the ratio is much less than the second term, the percent will be relatively small.

1 Divide seven by four to get the decimal equivalent of 1.75. Then, move the decimal point two places to the right to get the percent equivalent: 175%.

Because $\frac{3}{4}$ is a common fraction, students can write $\frac{7}{4}$ as the mixed number $1\frac{3}{4}$ and find the decimal equivalent for both one and $\frac{3}{4}$ without dividing.

2 Divide three by 500. Multiply the numerator and denominator by one-fifth which is three-fifths over one hundred or three-fifths percent. An alternate method is to divide 500 by three and get 0.006. Move the decimal point two places to the right to find its percent equivalent: 0.6%.

Because the denominator is a multiple of 100, students can divide the numerator and denominator by five to find the percent because a percent is part of 100. Dividing by five is the same as multiplying by $\frac{1}{5}$.

Additional Examples

1. Write as a percent.

4 to 200

Write the ratio as a fraction: $\frac{4}{200}$.

Write the fraction with a denominator of 100 by dividing both the numerator and denominator by two: $\frac{2}{100}$.

Because a percent is out of 100, the fraction is equivalent to 2%.

2. Write as a percent.

$$\mathbf{\frac{7}{5}}$$

The ratio is already a fraction. Divide seven by five: 1.4.

Write the decimal as a percent: 140%.

Alternatively, write the fraction as a mixed number and use knowledge of common fraction/decimal equivalents:

$$\frac{7}{5} = 1\frac{2}{5} = 1 + \frac{2}{5} = 100\% + 40\% = 140\%.$$

Expand Their Horizons

In this subtopic, students solve word problems by finding the percent of a number. To find the percent of a number, multiply the number by the percent. The percent can be changed to either a fraction or decimal.

The percent is best changed to a fraction when the percent/fraction equivalent is already known and/or when the denominator of the fraction is a multiple of the number, because these numbers will divide out upon simplification.

Remind students that a percent is changed to a decimal by moving the decimal point two places to the left. When multiplying decimals, multiply as with whole numbers and then, place the decimal point so that the product has the same number of decimal places as the factors.

Students can use one method to solve the problem and the other to check their answer. Again, encourage students to estimate their answers to check for reasonableness.

3 Multiply the number of computers by the decimal equivalent, 0.28. Alternatively, the solution using a fraction equivalent is $\frac{\cancel{28}^{14}}{\cancel{100}_{\cancel{2}_1}} \times \frac{\cancel{250}^{5}}{1} = 14 \times 5 = 70$. Students may also use their knowledge of the meaning of percents to solve this problem. They could say, since 28% means 28 out of every 100 and there are two 100's in 250, that would make 56 plus 14 more for the other 50, making a total of 70.

4 One-fourth percent can be converted to 0.25% and then to its decimal equivalent 0.0025. Then, multiply 6,400 by 0.0025 to get 16. However, since four divides evenly into 6,400, it makes sense to convert $\frac{1}{4}$% to its fractional equivalent by dividing $\frac{1}{4}$ by 100. To divide, multiply the dividend by the reciprocal of the divisor: $\frac{1}{4} \times \frac{1}{100} = \frac{1}{400}$.

Students may choose to look at $\frac{\frac{1}{4}}{100}$ as a complex fraction and multiply the numerator and denominator by four to change the fraction to $\frac{1}{400}$. Tell students that both strategies are valid.

Additional Examples

1. **Rachel earned \$450 by babysitting over the summer and put 60% of this money into a savings account. How much of her babysitting money did she put into her savings account?**

Find 60% of 450.

60% of 450

$$\frac{3}{\cancel{5}_1} \times \frac{\cancel{450}^{90}}{1}$$

270

Rachel put \$270 into her savings account.

2. **There are 600 rooms in a hotel, and $\frac{1}{2}$% of them are vacant. How many of the rooms are vacant?**

Find $\frac{1}{2}$% of 600.

$$\frac{1}{2}\% = \frac{\frac{1}{2}}{100} = \frac{1}{2} \times \frac{1}{100} = \frac{1}{200}$$

$$\frac{1}{\cancel{200}_1} \times \frac{\cancel{600}^3}{1} = 3$$

There are three vacant rooms.

Subtopic 3 — **Proportions**

Expand Their Horizons

In this subtopic, students are introduced to proportions. A proportion is an equation stating that two ratios are equal. The ratios are equivalent fractions—they represent the same value. For example, in the ratio $\frac{1}{2} = \frac{2}{4}$, each ratio is equivalent to 50%.

The ratio $\frac{1}{2} = \frac{2}{4}$ can also be written as 1 : 2 = 2 : 4. When written the latter way, it is easy to identify the means and extremes by remembering that *means* and *middle* both begin with *m*. The two's are the means because they are the middle terms. The extremes are the external or outer terms, which are one and four. Show that *extremes* and *external* both begin with *e*.

In a proportion, the product of the means equals the product of the extremes (called the cross products). This is one way to check if two ratios are in proportion. Another way to check is to simplify each ratio. If they are the same, the ratios are in proportion.

Common Error Alert:

Students may not recognize proportional ratios when they cannot find a whole number which they can multiply or can divide one part of one ratio by, to make that part of the other ratio. For example, in the proportion $\frac{2}{12} = \frac{3}{18}$, there is no whole number to multiply two by to get three. However, the equation is a proportion because both ratios simplify to $\frac{1}{6}$. Also, the products of the means and extremes are both 36.

In the lesson, a problem is solved showing that the ratios of square to round beads on two necklaces are the same. Point out that this does not mean that both necklaces necessarily have the same number of beads. One necklace can have 30 square beads and 70 round beads, while the other has 60 square beads and 140 round beads. Both would have a square to round bead ratio of three to seven, but the first would have a total of 100 beads while the second would have a total of 200 beads.

5 The ratios are proportional because the cross products are both 60. Also, the second ratio simplifies to $\frac{4}{5}$.

Additional Examples

1. Are $\frac{4}{21}$ and $\frac{3}{7}$ in proportion?

The product of the extremes is $4 \times 7 = 28$.
The product of the means is $21 \times 3 = 63$.

No, the ratios are not in proportion.

2. Are $\frac{8}{28}$ and $\frac{14}{49}$ in proportion?

Both ratios simplify to $\frac{2}{7}$.

$$\frac{8 \div 4}{28 \div 4} = \frac{2}{7}$$

$$\frac{14 \div 7}{49 \div 7} = \frac{2}{7}$$

Yes, the ratios are in proportion.

Look Beyond

Writing and solving a proportion is a procedure that can be used to solve many types of problems, including percent problems. Students will see how to use the percent proportion $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ to find the percent of a number (part), to find what percent one number is of another, and to find a number when a percent of it is given (whole). In geometry, problems involving similar figures, indirect measurement, and scale drawings are also solved with proportions.

Connections

Journalists often use percents and ratios when reporting news stories. For example, a sports writer may report a team's ratio of wins and losses or their ratio of wins to the total number of games played. This helps make comparisons among many teams much easier. A political writer may report the percent of people who voted for, or who say they will vote for, a certain candidate. A story about population change may report that the population this year is 110% of what it was a decade ago. An understanding of percents and ratios is necessary to fully understand the content of a newspaper.