Numbers and Operations

Module 6

Computational Fluency of Fractions

Lesson 6
Dividing Fractions

Teacher Notes 6.6

Objectives

- Model division of fractions using diagrams and/or illustrations of manipulatives.
- Develop and use algorithms for dividing fractions.

Prerequisites

Dividing whole numbers

Multiplying fractions

Simplifying fractions

Vocabulary

Fraction (4.1)
Product (2.3)
Improper fraction (4.1)
Mixed number (4.2)
Denominator (4.1)
Numerator (4.1)
Dividend (2.4)
Divisor (2.4)
Quotient (2.4)
Reciprocal
Multiplicative inverse
Common denominator (6.1)
Least Common Denominator (6.3)

Get Started

- Ask students how many halves are in one whole. **2**
- Ask how many thirds are in one whole. **3**
- Have volunteers come to the board to model each of the situations.
  Possible models:

$\frac{1}{2}$	$\frac{1}{2}$

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

- Have students use those models to answer the following:
  How many halves are in two wholes? **4**
  How many thirds are in two wholes? **6**

- Tell students to use what they see to find how many fourths are in 10 wholes. **40**
  Have students discuss how they found their answers.
  Possible answer: One whole has four fourths, so I multiplied four by 10 to get 40.

Subtopic 1 Dividing Using Models and the Invert-and-Multiply Algorithm

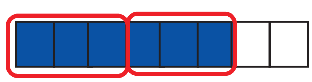
Expand Their Horizons

In this subtopic, students divide fractions and mixed numbers by positive whole numbers. They first learn to divide using models. The dividend is modeled and then is divided into the number of groups given by the divisor. The size of each group is the quotient.

For example, to find $\frac{3}{4} \div 2$, first model $\frac{3}{4}$.

Because three sections cannot be divided by two equally, divide each section in half.

To divide by two, divide the six sections into two equal groups.



Each group is $\frac{3}{8}$, so $\frac{3}{4} \div 2 = \frac{3}{8}$.

After modeling, students learn that dividing by a whole number is the same as multiplying by its reciprocal. The reciprocal of a whole number is one divided by that number. A reciprocal can also be called the multiplicative inverse; because under a given operation with a given number, an inverse makes the answer equal to the identity for that operation. Because the identity for multiplication is one, the multiplicative inverse of a number is the number that when multiplied with the given number has a product of one.

For students struggling with the basic concepts, relate the division to money. Ask students how much five people would get if they divided 50 cents equally. They should answer 10 cents without difficulty. Now show that 50 cents = $\frac{1}{2}$ because it is one-half of a dollar. Fifty cents divided by five is the same as $\frac{1}{2} \div 5$. Using the rules, $\frac{1}{2} \div 5 = \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$. The fraction $\frac{1}{10}$ is the same as 10 cents because it is one-tenth of a dollar.

1

Model the dividend by shading five of eight equal parts. When divided into five groups, each group is one square, which represents $\frac{1}{8}$.

2

Model $1\frac{4}{5}$. So that each is divided into equal-sized parts, divide the whole into fifths. Nine squares are shaded. When divided into three equal groups, each group has three squares, or $\frac{3}{5}$ which represents $\frac{3}{5}$ of an hour that Pedro can study for each test.

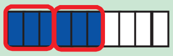
Additional Examples

1. Use a model to divide $\frac{3}{5}$ into two equal groups.

Model $\frac{3}{5}$.

Three cannot be divided evenly by two, so divide each square in half.

Divide the six squares into two groups.



Each group is $\frac{3}{10}$.

2. Find $\frac{12}{17} \div 4$ without models.

The reciprocal of four is $\frac{1}{4}$. Multiply the dividend by $\frac{1}{4}$.

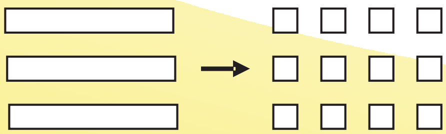
$$\frac{{}^{3}\cancel{12}}{17} \times \frac{1}{\cancel{4}_{1}} = \frac{3}{17}$$

Subtopic 2

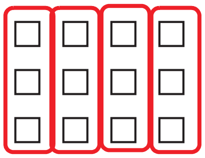
Dividing Using Models and the Common Denominator Algorithm

Expand Their Horizons

In this subtopic, students divide by fractions rather than by whole numbers. When modeling whole numbers divided by fractions, model the number of wholes and then divide each whole based on the denominator of the divisor. For instance, to find $3 \div \frac{3}{4}$, divide each of three wholes into fourths.



Since each square is $\frac{1}{4}$, to divide by $\frac{3}{4}$ means to group the squares so that each group has three squares. The quotient is the number of groups, in this case, four.



$$3 \div \frac{3}{4} = 4$$

To divide a fraction by a fraction with a model, again model the dividend first. If the fractions have unlike denominators, divide the model into smaller parts and/or find an equivalent fraction for the divisor. Then, find the number of groups.

In the lesson on dividing by decimals, students learned they could multiply the dividend and divisor by the same power of 10 and the quotient would not change. Students will apply this same strategy to dividing fractions by multiplying the dividend and the divisor by the reciprocal of the divisor. Because a number and its reciprocal have a product of one, resulting in a divisor of one, the dividend will be multiplied by the reciprocal of the divisor. In other words, the reciprocal of $\frac{m}{n}$ is $\frac{n}{m}$ because $\frac{m}{n} \times \frac{n}{m} = 1$. This is the most common method of dividing fractions and is often called the Invert-and-Multiply Method.

Common Error Alert:

Students may take the reciprocal of the dividend rather than the divisor. Show students with examples that the two methods lead to different answers.

Correct:

$$\frac{3}{8} \div \frac{1}{4} = \frac{3}{8} \times \frac{4}{1} = \frac{12}{8} = 1\frac{4}{8} = 1\frac{1}{2}$$

Incorrect:

$$\frac{3}{8} \div \frac{1}{4} = \frac{8}{3} \times \frac{1}{4} = \frac{8}{12} = \frac{2}{3}$$

Another method is the Common Denominator Method. Students write each fraction so that they have a common denominator and then just divide the numerators. If the numerators do not divide evenly, then form a fraction, either proper or improper. If the fraction is improper, rewrite it as a mixed number. This method is best used when a common denominator is easily spotted.

3

Model the dividend by shading 15 of 16 squares. The divisor is not in sixteenths, so write an equivalent fraction for it: $\frac{6}{16}$. With just the 15 shaded squares, form groups so that six squares are in each group. Count the number of groups: two full groups and one-half of a group.

Additional Examples

1. Divide.

$$3\frac{1}{2} \div 1\frac{3}{4}$$

Write the mixed numbers as improper fractions.

$$\frac{7}{2} \div \frac{7}{4}$$

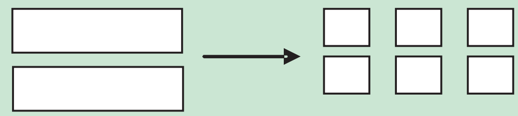
Invert the divisor and multiply.

$$\frac{{}^{1}\cancel{7}}{{}_{1}\cancel{2}} \times \frac{\cancel{4}^{2}}{\cancel{7}_{1}} = \frac{2}{1} = 2$$

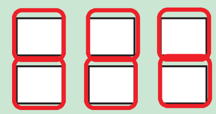
2. Model.

$$2 \div \frac{1}{3}$$

Draw two wholes, each divided into thirds.



Make groups, each with $\frac{1}{3}$, or one square.



There are six groups. The quotient is six.

Look Beyond

In algebra, equations such as $\frac{2}{3}x = 12$ can be solved by either dividing both sides of the equation by $\frac{2}{3}$ or by multiplying both sides of the equation by the reciprocal of $\frac{2}{3}$, or $\frac{3}{2}$. Even equations that may not first appear to have a fraction can be solved by dividing fractions. For instance, $\frac{m}{5} = 3$ is the same as $\frac{1}{5}m = 3$.

Connections

Chefs and caterers divide fractions when they need to feed less people than for what their recipe was designed for. For instance, suppose a recipe feeds 20 people and a chef is cooking for a 12-person party. The chef can divide the amount of each ingredient by $1\frac{2}{3}$ or multiply by $\frac{3}{5}$ which is the reciprocal of $1\frac{2}{3}$. The original amounts may be whole numbers, mixed numbers, or fractions. By dividing the recipe, the chef can reduce the amount of food wasted and also save money.