Numbers and Operations

Module 6

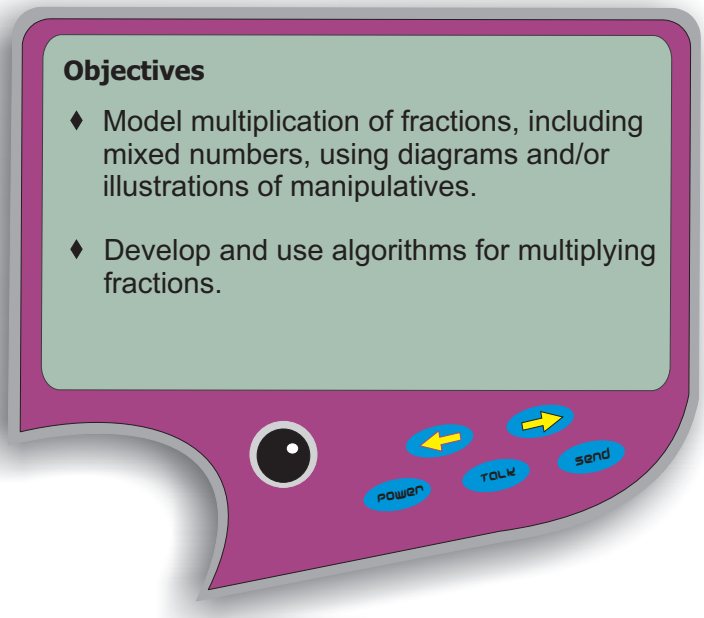
Computational Fluency of Fractions

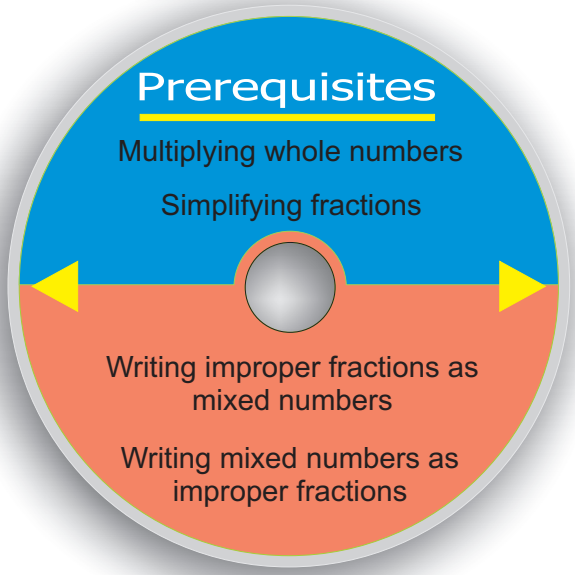
Lesson 5
Multiplying Fractions

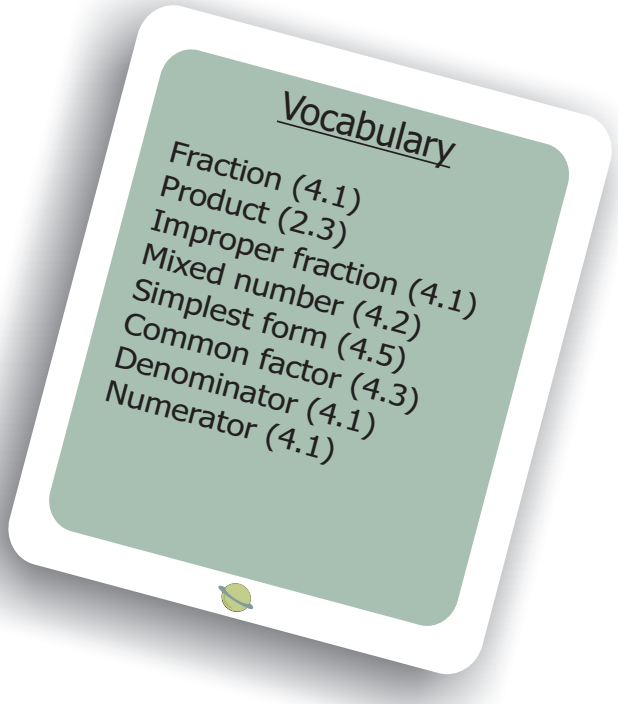
Teacher Notes 6.5

Objectives

- Model multiplication of fractions, including mixed numbers, using diagrams and/or illustrations of manipulatives.
- Develop and use algorithms for multiplying fractions.







Get Started

- Write $\frac{1}{6}+\frac{1}{6}$ on the board. Ask the students for the sum both before and after simplifying. $\frac{2}{6}, \frac{1}{3}$
- Write $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$ on the board. Ask the students for the sum both before and after simplifying. $\frac{3}{6}, \frac{1}{2}$
- Write $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$ on the board. Ask the students for the sum both before and after simplifying. $\frac{4}{6}, \frac{2}{3}$

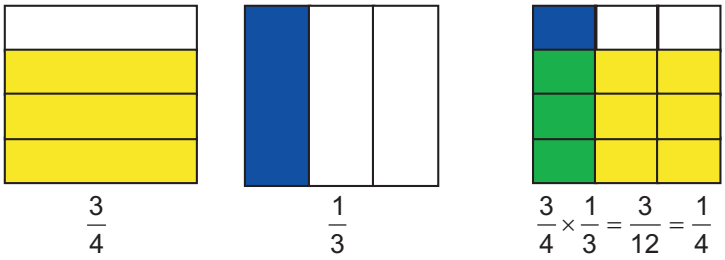
- Ask students how to write $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$ as a multiplication problem. **$4\times\frac{1}{6}$**

- Have students study the multiplication problem and the answer to the related addition problem. Then ask them to solve $20\times\frac{1}{6}$. Have students explain their reasoning. **$3\frac{1}{3}$: Possible answer: There are 20 groups of $\frac{1}{6}$ and $\frac{6}{6}$ makes one whole, so $\frac{20}{6}$ makes three wholes with $\frac{2}{6}$ left over.**

Subtopic 1 Multiplying Fractions with and Without Models

Expand Their Horizons

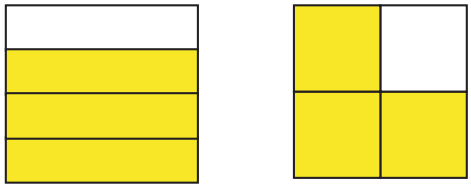
In this subtopic, students first see how models can be used to multiply fractions, which is similar to modeling the product of decimals; students model the first fraction horizontally and the second fraction vertically. The product is the overlapping region. The product of $\frac{3}{4}\times\frac{1}{3}$ is shown below.



$\frac{3}{4}$ $\frac{1}{3}$ $\frac{3}{4}\times\frac{1}{3}=\frac{3}{12}=\frac{1}{4}$

This method only works when the models are divided into "strips." One factor is modeled horizontally and the other vertically.

$\frac{3}{4}$ Correct: $\frac{3}{4}$ Incorrect:



After working with models, the students will multiply fractions by multiplying the numerators, multiplying the denominators, and then simplifying. Remind students that a whole number can be written as a fraction by writing the number over one. The product of two fractions may be a fraction, a whole number, or a mixed number.

1

The product of the numerators is 70, and the product of the denominators is 105. The greatest common factor is 35. If students do not see the GCF of 35, they can first divide out five and then seven, or vice-versa.

After seeing that $\frac{2}{3}$ of John's video games are basketball games and knowing the total number of video games is 42, students can multiply to find *how many* of his video games are basketball games. John has 28 basketball video games.

2

The whole number 12 is equivalent to $\frac{12}{1}$. Multiply the numerators: 36; multiply the denominators: four; and simplify. The product is the whole number nine. Kody walks nine miles.

Encourage students to estimate answers before multiplying. In Lesson Notes Problem Two, students should see that Kody walks almost one mile every day, so the final answer should be close to, but less than, 12 miles. Students might also see that $\frac{1}{4}$ of 12 miles is three; therefore, $\frac{3}{4}$ is equal to nine miles.

Additional Examples

1. At Mavis's Diner, $\frac{3}{4}$ of the sandwiches were made on white bread. Of those on white bread, $\frac{6}{7}$ were made with turkey. What fraction of all the sandwiches at the diner used turkey on white bread?

$$\frac{3}{4}\times\frac{6}{7}$$

Multiply the numerators: 18.
Multiply the denominators: 28.

$$\frac{3}{4}\times\frac{6}{7}=\frac{18}{28}$$

Divide out the common factor of two.

$$\frac{18\div 2}{28\div 2}=\frac{9}{14}$$

$\frac{9}{14}$ used turkey on white bread.

2. Multiply.

$$35\times\frac{3}{5}$$

Write 35 as a fraction: $\frac{35}{1}$.

$$\frac{35}{1}\times\frac{3}{5}=\frac{105}{5}$$

Divide 105 by five: $\frac{105}{5}=21$.

Subtopic 2 — Simplifying Fractions Before Multiplying

Expand Their Horizons

In this subtopic, students are shown that they can simplify before multiplying the numerators and denominators. A common factor can be divided from any numerator and any denominator; they can be in the same fraction or in different fractions.

Show students that they can divide a common factor out of a number that already had a common factor removed. For instance, in the problem below, three was divided out of six and nine. Now, three can be divided out of both the numerator and denominator in the second fraction.

$$\frac{\overset{2}{\cancel{6}}}{5} \times \frac{3}{\cancel{9}_{3}} \longrightarrow \frac{\overset{2}{\cancel{6}}}{5} \times \frac{\cancel{3}^{1}}{\cancel{9}_{\cancel{3}_{1}}} = \frac{2}{5} \times \frac{1}{1} = \frac{2}{5}$$

Common Error Alert:

Students will sometimes "lose" their new numerators and denominators in a rush to divide out common factors. Tell students to make their crossing out marks neatly, so they do not cross out what is still needed. They may also wish to circle the numerators and denominators before multiplying.

It is alright if students do not catch all of the simplifying before they multiply. They can still simplify their final answer. Any simplifying done before multiplying can make the problem easier to solve.

3

Common factors can be divided out of nine and 27, as well as eight and 20. Then, multiply one and two for the numerator of the product and five and three for the denominator of the product. Of the students, $\frac{2}{15}$ eat breakfast at school. The problem continues by determining the number of students who eat breakfast at school from the total number of students. Point out that the numerators and denominators were not factored before multiplying, but it did not make a difference in the answer since the fraction was simplified after multiplying.

4

A common factor of three can be divided out from nine and 12. In the product, the numerator is eight times four, and the denominator is three times 13. The fraction of students who stay after school to play sports is $\frac{32}{39}$.

Common Error Alert:

Students may divide out common factors in two numerators or two denominators. In Lesson Notes Problem Four, students who may have answered $\frac{2}{39}$ divided a common factor of four from the eight and four in the numerators. Stress that one of the numbers has to be a numerator and one has to be a denominator.

Additional Examples

1. In an art store, $\frac{5}{6}$ of the frames are made of wood. Of those wood frames, $\frac{12}{25}$ display a photo. What fraction of all the frames displays a photo and is made of wood?

$$\frac{12}{25} \times \frac{5}{6}$$

Five and 25 have a common factor of five, and 12 and six have a common factor of six.

$$\frac{\cancel{12}^{2}}{\cancel{25}_{5}} \times \frac{\cancel{5}^{1}}{\cancel{6}_{1}}$$

Multiply the numerators. Multiply the denominators.

$$\frac{1}{1} \times \frac{2}{5} = \frac{2}{5}$$

$\frac{2}{5}$ of the frames are wood and display photos.

2. Multiply.

$$\frac{3}{8} \times \frac{12}{17}$$

Divide a common factor of four from eight and 12.

$$\frac{3}{\cancel{8}_{2}} \times \frac{\cancel{12}^{3}}{17}$$

Multiply the numerators. Multiply the denominators.

$$\frac{3}{2} \times \frac{3}{17} = \frac{9}{34}$$

Subtopic 3 — Multiplying with Mixed Numbers

Expand Their Horizons

In this subtopic, students multiply mixed numbers by whole numbers, fractions, and other mixed numbers. Once the mixed numbers are rewritten as improper fractions, they can be multiplied as usual.

5

Change the first factor to an improper fraction and write the whole number over one. Divide out a common factor of three and multiply the numerators and denominators. Remind students to use estimation to check the accuracy or the reasonableness of their answer.

Additional Examples

1. Multiply.

$$1\frac{2}{9} \times \frac{27}{28}$$

Rewrite the mixed number as an improper fraction and divide out the common factor of three.

$$\frac{11}{\cancel{9}_{1}} \times \frac{\cancel{27}^{3}}{28}$$

Then, multiply across and write the answer as a mixed number.

$$\frac{11}{1} \times \frac{3}{28} = \frac{33}{28} = 1\frac{5}{28}$$

2. For a project, Mica will need 24 sections of rope, each $3\frac{1}{6}$ feet long. How many feet of rope will Mica need altogether?

Write the mixed number as an improper fraction and write 24 over one.

$$3\frac{1}{6} \times 24 = \frac{19}{6} \times \frac{24}{1}$$

Divide out the common factor of six and multiply.

$$\frac{19}{\cancel{6}_{1}} \times \frac{\cancel{24}^{4}}{1} = \frac{19}{1} \times \frac{4}{1} = 76$$

Mica will need 76 feet of rope.

Look Beyond

Students will soon divide fractions and will learn that dividing by a number is the same as multiplying by its reciprocal. One of the algorithms for dividing fractions is to write the reciprocal of the divisor and to multiply. Therefore, if students choose to use this algorithm, they will need to be proficient in multiplying fractions.

Connections

Multiplication of and by fractions is used in many different careers, including those in the medical field. Nurses learn the "nursing rule" which consists of "the drug prescribed divided by the dose per measure times the number of measures."