# Numbers and Operations

★ **Module 6** ★

## Computational Fluency of Fractions

**Lesson 4
Adding and Subtracting
Mixed Numbers**

**Objectives**

♦ Model addition and subtraction of mixed numbers using diagrams and/or illustrations of manipulatives.

♦ Develop and use algorithms to add and to subtract mixed numbers.

POWER   TALK   SEND

## Prerequisites

Adding and subtracting fractions with like and unlike denominators

Identifying parts of a fraction

Modeling fractions

Writing improper fractions as mixed numbers

Simplifying fractions

## Vocabulary

Improper fraction (4.1)
Mixed number (4.2)
Simplest form (4.5)
Least Common Denominator (6.3)
Least Common Multiple (4.4)
Unlike denominators (6.2)
Common denominator (6.1)
Minuend (2.2)
Subtrahend (2.2)
Addend (2.1)
Whole number (3.1)
Equivalent fraction (4.1)

## Get Started

- On the board, draw one group of two pizzas and one pizza slice and another group of one pizza and four pizza slices. Have students add the pizzas and then the pizza slices.
  **Three pizzas and five slices**

- Erase those pizzas and now draw one group of three pizzas and six slices and another group of four pizzas and four slices. Have students add the pizzas and the pizza slices.
  **Seven pizzas and 10 slices**

- Ask students if they can think of another way to write the answer, knowing that eight slices will form one whole pizza. Have them explain their reasoning.
  **Possible answer: Because eight slices make one pizza which would be added to the seven whole pizzas, there would be eight pizzas with two slices left over.**

**6.4A**

- Tell students that the pictures on the board were models of the addition of $3\frac{3}{4}+4\frac{1}{2}$ and that in the lesson they will learn to add and to subtract mixed numbers with and without models.

## Expand Their Horizons

In this subtopic, students begin adding mixed numbers by using fraction bars. They model each addend and then combine the models. When modeling fractions with unlike denominators, they will need to divide parts of a model so that parts in both models are the same size. For example, to add $1\frac{3}{4}$ and $1\frac{1}{2}$, each half in the second addend is divided in half so that both models have parts in fourths.

$1\frac{3}{4}$          $1\frac{1}{2}$          becomes          $1\frac{3}{4}$          $1\frac{2}{4}$
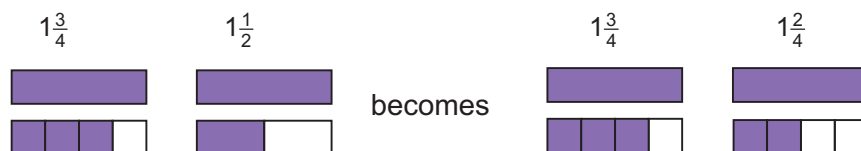
When the models are combined, one fourth, from the second addend, moves to the first model, so the sum has three wholes and one fourth.

$3\frac{1}{4}$
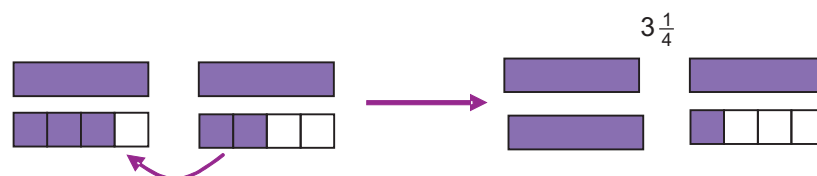
After modeling sums of mixed numbers, students add mixed numbers without models. As with adding fractions, the fraction parts of the mixed numbers must have a common denominator. Therefore, the first step is writing equivalent fractions with a common denominator, if needed. This does not affect the whole number parts of the mixed numbers. Once the fractions have a common denominator, the fractions are added. Then the whole numbers are added. Combine the two sums.

**Common Error Alert:**

**Students may add the fraction parts and forget to add the whole number parts. This is most likely to occur when they have to find equivalent fractions with a common denominator. Remind students always to look back at the original problem and to see if their answer is reasonable. An estimate of the sum will alert them if they have made this error.**

**1** Because the two lengths have a common denominator, add the numerators. The sum of the numerators is six. The common denominator is five, so the sum of the fractional parts is $\frac{6}{5}$ or $1\frac{1}{5}$ or $1+\frac{1}{5}$. The sum of the whole number parts is six. Combine the sums: $6+1+\frac{1}{5}=7\frac{1}{5}$.

Most students will not have difficulty seeing immediately that $1\frac{1}{5}+6=7\frac{1}{5}$ and will not separate $1\frac{1}{5}$ into $1+\frac{1}{5}$. Nevertheless, remind them that they are adding whole number parts and fraction parts separately.

**2** Rewrite the fractional parts as equivalent fractions with a common denominator of 10. The sum of the fractions is $\frac{13}{10}$, and the sum of the whole numbers is nine. Rewrite $\frac{13}{10}$ as $1\frac{3}{10}$ and add nine: $10\frac{3}{10}$.

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### Additional Examples

1. **For a remodeling project, Terrence bought $1\frac{1}{4}$ pounds of nails and $2\frac{3}{4}$ pounds of screws. How many total pounds of nails and screws did he buy?**

   Find the sum of the fractional parts: $\frac{4}{4}$.

   Rewrite this improper fraction as a whole number: 1.

   Find the sum of the whole numbers: 3. Combine the sums: 1 + 3 = 4.

   Terrence bought four pounds of nails and screws.

2. **Sam worked $2\frac{1}{4}$ hours overtime on Monday and $1\frac{2}{5}$ hours overtime on Tuesday. How much total overtime did he work both days?**

   Rewrite both fractions with the LCD of 20.

   $$2\frac{1}{4} \rightarrow 2\frac{5}{20}$$
   $$+1\frac{2}{5} \rightarrow 1\frac{8}{20}$$

   Add the fractions; add the whole numbers.

   $$3\frac{13}{20}$$

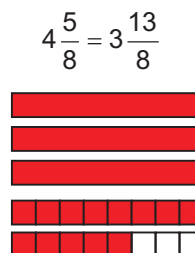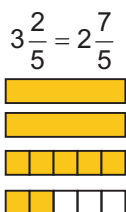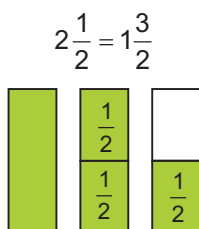   Sam worked $3\frac{13}{20}$ overtime hours.

## Expand Their Horizons

In this subtopic, students subtract mixed numbers, first by using models, and then by using the algorithm. Depending on the ability level of the students, teachers may wish to take some time to look at regrouping in models. Here are three examples to share with students.

$$2\frac{1}{2} = 1\frac{3}{2}$$

$$3\frac{2}{5} = 2\frac{7}{5}$$

$$4\frac{5}{8} = 3\frac{13}{8}$$

To subtract mixed numbers with models, students will model the minuend and then take away (by crossing out) the wholes and parts in the subtrahend. The remaining shaded areas represent the difference.

Subtracting mixed numbers without models uses the same rules as adding mixed numbers without models. The difference between the two is that in subtracting, students may need to regroup. They will always regroup when the minuend is a whole number. Another reason for regrouping is when the numerator in the subtrahend is less than the numerator in the minuend. Tell students not to regroup until the fractions have like denominators.

**Common Error Alert:**

**Students may automatically add 10 to the numerator when they regroup the minuend, which is an automatic response from using the base 10 decimal system. Remind students that the denominator tells how many parts make a whole, and because they are regrouping a whole, they should add the value of the denominator to the numerator.**

| **Incorrect:** | **Correct:** |
|---|---|
| $3\frac{5}{8} = 2\frac{15}{8}$ | $3\frac{5}{8} = 2\frac{8+5}{8} = 2\frac{13}{8}$ |

Many students prefer and make fewer errors when subtracting vertically. Remind students they can rewrite any problem given to them in the vertical format.

**6.4D**

**3** The fractions have like denominators already. The first step is to see if regrouping is needed. In the numerators, three cannot be subtracted from one, so rewrite $3\frac{1}{4}$ as $2\frac{5}{4}$. Now subtract the fractions, which leave $\frac{2}{4}$, and the whole numbers, which leave 1. The difference is $1\frac{2}{4}$, which simplifies to $1\frac{1}{2}$.

**Common Error Alert:**

**Students may forget to subtract one from the whole number part when they regroup. Have them cross out the whole number and write one less before writing the fractional part. Students who answered $2\frac{1}{2}$ to Lesson Notes Problem Three may have made this error.**

**4** The minuend is a whole number, so it is renamed as $4\frac{5}{5}$. Subtract fractions and whole numbers as usual.

**Common Error Alert:**

**Students may attempt to subtract mixed numbers from whole numbers by using mental math. In so doing, they increase their chances of not subtracting one from the whole number part of the minuend. For example, they may say the difference between 5 and $3\frac{2}{5}$ is $2\frac{3}{5}$. Have them check their work by finding the sum. In this case, $2\frac{3}{5} + 3\frac{2}{5}$ is greater than five, so the answer to the subtraction problem is wrong.**

**5** First, rewrite the fractions with a common denominator. The LCD is 12. Nine cannot be subtracted from two, so rewrite $9\frac{2}{12}$ as $8\frac{14}{12}$. Then, subtract the parts of the mixed numbers.

**Additional Examples**

1. **The total weight a trailer can hold is $5\frac{3}{4}$ tons. A partial load of $2\frac{1}{4}$ tons has already been loaded on the trailer. How many additional tons can be loaded before reaching maximum capacity?**

   The problem is $5\frac{3}{4} - 2\frac{1}{4}$.

   Subtract the fractions, which leave $\frac{2}{4}$.

   Subtract the whole numbers, which leave 3.

2. **Tyrese bought $9\frac{1}{3}$ cubic yards of gravel to cover his driveway. The company delivered $4\frac{1}{2}$ cubic yards on the first trip. How much gravel do they still need to deliver?**

   The problem is $9\frac{1}{3} - 4\frac{1}{2}$.

   Rewrite each fraction with the least common denominator, 6.

   $$9\frac{2}{6} - 4\frac{3}{6}$$

**6.4E**

Combine the parts and simplify.

$$3\frac{2}{4} = 3\frac{2 \div 2}{4 \div 2} = 3\frac{1}{2}$$

$3\frac{1}{2}$ additional tons can be loaded.

Regroup the minuend. Subtract the numerators.

$$8\frac{8}{6} - 4\frac{3}{6}$$

Subtract the fractions. Subtract the whole numbers.

$$8\frac{8}{6} - 4\frac{3}{6} = 4\frac{5}{6}$$

The company still needs to deliver $4\frac{5}{6}$ cubic yards of gravel.

## Look Beyond

Because fractions occur so often in measurement, students will need to be able to add fractions and mixed numbers to answer geometry problems such as finding the perimeter of a figure whose side lengths are not whole numbers. They will need to subtract fractions and mixed numbers when given the perimeter and when asked to find a missing side length.

## Connections

Automotive technicians, plumbers, and electricians usually charge labor costs in addition to the cost of materials. Depending on the company's policy, the cost of labor can be rounded to the nearest half or quarter hour. Therefore, these workers keep track of the time they spend on each job.

Suppose a plumber visited a certain site three times and the time spent working during each visit was $2\frac{1}{4}$ hours, $1\frac{1}{2}$ hours, and $1\frac{3}{4}$ hours. The plumber would need to be able to add those values to properly charge the customer for the work completed.

**6.4F**