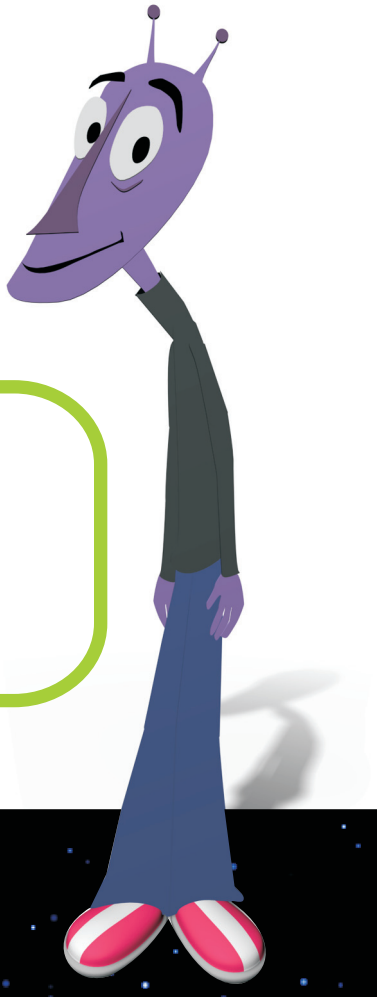
Numbers and Operations

Module 6

Computational Fluency of Fractions

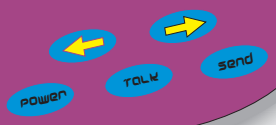


Lesson 2
Adding Fractions with Unlike Denominators

Teacher Notes 6.2

Objectives

- Find equivalent fractions.
- Model addition of fractions with unlike denominators using diagrams and/or illustrations of manipulatives.
- Develop and use algorithms to add fractions with unlike denominators.



Prerequisites

Adding fractions with like denominators

Modeling fractions

Finding the least common multiple of two numbers

Writing improper fractions as mixed numbers

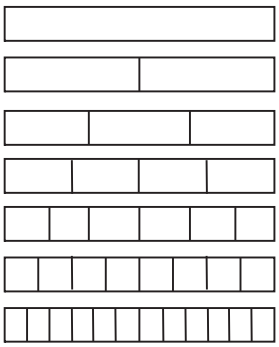
Simplifying fractions

Vocabulary

Fraction (4.1)
Mixed number (4.2)
Numerator (4.1)
Denominator (4.1)
Like denominators (6.1)
Common denominator (6.1)
Improper fraction (4.1)
Addend (2.1)
Common factor (4.3)
Least common multiple (4.4)
Simplest form (4.5)
Sum (2.1)
Equivalent fraction (4.1)
Unlike denominators

Get Started

- Draw the following fraction bars on the board (unit, halves, thirds, fourths, sixths, eighths, twelfths).
- Have volunteers write fractions inside the rectangles.
- Use the models to show that $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$ because two eighths and one fourth cover the same amount of one unit.
- Have students use the models to make other addition facts. Encourage multiple addends as well as unlike denominators, such as $\frac{1}{4}+\frac{1}{4}+\frac{1}{2}=1$ and $\frac{1}{12}+\frac{1}{12}+\frac{1}{6}=\frac{1}{3}$.



Expand Their Horizons

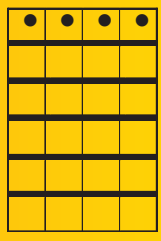
In this subtopic, students use egg cartons to model addition of fractions with unlike denominators. They see how to add fractions using cartons with dimensions of 3×4. These can be divided equally into halves, thirds, fourths, sixths, and twelfths. Draw five 3×4 egg cartons on the board and have students divide each model into each of the possible five ways. Show that there may be more than one way to divide some of the fractions. For example, to make halves, they can divide the egg carton either vertically or horizontally.

Students will also model with egg cartons with dimensions of 6×4. These can be divided into halves, thirds, fourths, sixths, eighths, twelfths, and twenty-fourths. Teachers may wish to draw seven 6×4 models on the board and have students divide each model into each of the seven ways.

When dividing models into their required number of parts, have students use bold lines or colored markers to help make it clear which lines show how many equal parts the carton is being divided into versus how many squares are in each part.

Common Error Alert:

Students may simply use the number of dots in a model as the denominator. That is, they may interpret the following model as $\frac{1}{4}$ because there are four dots.



Remind students that the number of parts, six, is the denominator, and one of those six parts is completely filled in, so the model represents $\frac{1}{6}$.

Use the models to discuss equivalent fractions. To display $\frac{3}{8}$, divide a 6×4 model into eight equal parts. Each part has three squares. It takes nine eggs to fill the three parts. This is nine out of 24 squares, so $\frac{3}{8}$ is equivalent to $\frac{9}{24}$.

Similar to modeling fractions with like denominators, students model each fraction, count the number of eggs in both models, draw a new model with that number of eggs, and then find what fraction of the new model is filled. When the sum is greater than one, additional egg cartons are drawn.

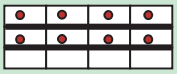
1 To model $\frac{5}{6}$, divide the egg carton into six equal parts. Each part has two squares. It takes 10 eggs to fill the five parts. To model $\frac{1}{4}$, divide the carton into four equal parts, each with three squares. It takes three eggs to fill one part. This is a total of 13 eggs, which fill one whole carton and $\frac{1}{12}$ of another.

2 To model the first addend, divide the carton into three equal parts. Each part has four squares. It takes eight eggs to fill two parts. To model the second addend, divide the carton into four equal parts, each with three squares. It takes nine eggs to fill three parts. The total of 17 eggs fills one whole carton and $\frac{5}{12}$ of another.

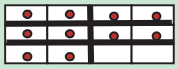
Additional Examples

1. Model $\frac{2}{3} + \frac{5}{6}$ using 3 × 4 egg cartons.

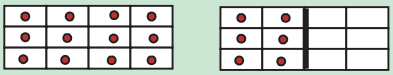
Divide one egg carton into three equal parts. Fill in two of those parts. Each part has four squares, so it takes eight eggs.



Divide a second carton into six equal parts. Fill in five of those parts. Each part has two squares, so it takes 10 eggs.



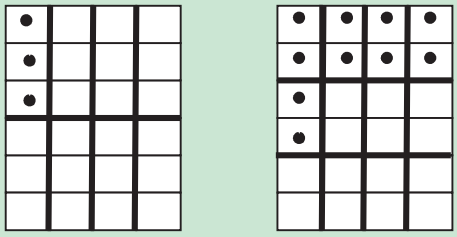
Count the eggs in both cartons: 18. Each carton holds 12 eggs, so the 18 eggs fill one carton completely with six eggs left over in another carton.



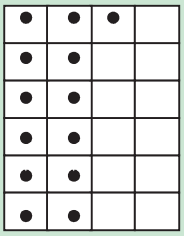
Six eggs make up half of a carton, so the sum is the mixed number $1\frac{1}{2}$.

2. Model $\frac{1}{8} + \frac{5}{12}$ using 6 × 4 egg cartons.

Divide one egg carton into eight equal parts and fill in one of those parts. This takes three eggs. Divide a second egg carton into 12 equal parts and fill in five of those parts. This takes 10 eggs.



Altogether, 13 eggs have been used and can be placed into one 6x4 egg carton. The sum is $\frac{13}{24}$.



Expand Their Horizons

In this subtopic, students add fractions with unlike denominators without models. They find the least common multiple (LCM) of the denominators and use that as the least common denominator (LCD). Students write equivalent fractions by multiplying both the numerator and denominator of one or both fractions by the same number.

In $\frac{3}{4}+\frac{2}{3}$, the LCM of four and three is 12. To obtain equivalent fractions with a denominator of 12, multiply both parts of the first addend by three and both parts of the second addend by four: $\frac{3\times3}{4\times3}+\frac{2\times4}{3\times4}$. The equivalent fractions, now with like denominators, are $\frac{9}{12}+\frac{8}{12}$.

Common Error Alert:

Students sometimes multiply the denominator of an addend by a certain number to make the LCD but then forget to multiply the numerator by the same number. Remind students that both the numerator and denominator must be multiplied by the same number to make equivalent fractions. The expression $\frac{3\times3}{4\times3}+\frac{2\times4}{3\times4}$ is the same as $\frac{3}{4}\left(\frac{3}{3}\right)+\frac{2}{3}\left(\frac{4}{4}\right)$ which is the same as $\frac{3}{4}(1)+\frac{2}{3}(1)$. The fractions are equivalent because they have been multiplied by a form of one.

3

To find the LCM of three and eight, list multiples of eight until one of them is also a multiple of three. The LCM is 24. Think of what each denominator needs to be multiplied by to make a product of 24. For the first addend, three times eight is 24, so multiply both the numerator and denominator of the first addend by eight. For the second addend, eight times three is 24, so multiply both the numerator and denominator of the second addend by three. Now the addends have like denominators. Add the numerators and keep the like denominator.

Additional Examples

1. **Add.**

$$\frac{7}{9}+\frac{1}{2}$$

2. **Tessa and Lavina bought a bucket of popcorn. Tessa ate $\frac{1}{6}$ of the popcorn and Lavina ate $\frac{2}{3}$ of the popcorn. What fraction of the bucket of popcorn did they eat?**

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The least common denominator of two and nine is 18, so multiply both parts of the first fraction by two and both parts of the second fraction by nine.

$$\frac{7}{9}+\frac{1}{2}$$
$$\frac{7\times2}{9\times2}+\frac{1\times9}{2\times9}$$
$$\frac{14}{18}+\frac{9}{18}$$

Add the numerators and keep the like denominator.

$$\frac{14+9}{18}$$
$$\frac{23}{18}$$
$$1\frac{5}{18}$$

The least common denominator of six and three is six. Keep the first fraction as is. Multiply both parts of the second fraction by two.

$$\frac{1}{6}+\frac{2}{3}$$
$$\frac{1}{6}+\frac{2\times2}{3\times2}$$
$$\frac{1}{6}+\frac{4}{6}$$

Add the numerators and keep the like denominator.

$$\frac{1+4}{6}$$
$$\frac{5}{6}$$

They ate $\frac{5}{6}$ of the bucket of popcorn.

Look Beyond

In the next lesson, students will subtract fractions with unlike denominators. They will begin with models, specifically fraction bars. To subtract fractions with unlike denominators using fraction bars, students will first model each fraction. Because the denominators will be different, the parts will not be the same size. They will then redraw one or both models so that both models have equal-size parts. This will allow parts to be taken away.

To subtract fractions with unlike denominators without a model, students will continue to use the skills they used in this lesson. They will find the least common multiple of the denominators and use that as the common denominator. They will keep the like denominator, subtract the numerators, and simplify if needed.

Connections

Athletes on a track team add fractions when they want to know the total distance they ran during practice. Because a track is one-fourth of a mile long, common training distances range from one-sixteenth of a mile (a quarter way around the track) to a half a mile (two laps around the track) and beyond. A runner who warmed up by running a half a mile, followed by sprinting several eighths of a mile, and finished with jogging a three-quarter mile cool down would add fractions with unlike denominators of two, four, and eight to find his or her total running distance for that day.