Numbers and Operations
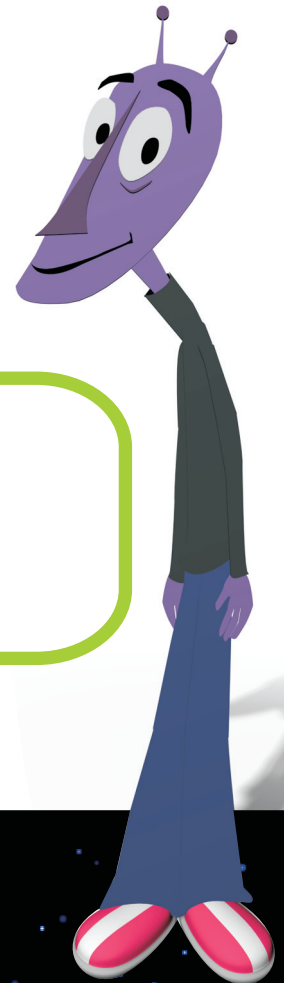
★ **Module 5** ★

Decimal Operations, Exponents, and Powers

Lesson 7
Scientific Notation

## Objectives

♦ Demonstrate an understanding of place value using powers of 10, and write numbers greater than one in scientific notation with and without appropriate technology.

♦ Convert between scientific notation and standard notation using numbers greater than one.

♦ Convert between scientific notation and standard notation using numbers from zero to one.

POWER   TALK   SEND
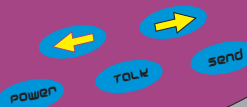
Prerequisites

Multiplying and dividing by powers of 10 by moving the decimal point

Writing powers of 10 in standard and exponential forms

Evaluating positive, negative, and zero exponents

Vocabulary

Scientific notation
Power of Ten
Integer (3.1)
Exponent (1.1)
Place value (4.2)
Base (1.1)
Decimal place (4.2)
Decimal point (4.2)
Nonzero integer (3.1)
Standard form of a number
Expanded form of a number

Instructional Alert

In order to adequately cover the content in this lesson, the lesson has been divided into two parts in an appropriate and logical place. Each part of the lesson may be presented in separate class periods or on separate days. In schools with extended class periods, teachers may present both parts as one lesson.

5.7A

Part One

Get Started

- Write the following exercises on the blackboard: Write in order from least to greatest 65,000,000,000,000,000,000,000; 9,300,000,000,000,000,000,000; and 729,000,000,000,000,000,000,000.
  9,300,000,000,000,000,000,000; 65,000,000,000,000,000,000,000; 729,000,000,000,000,000,000,000
  Write in order from greatest to least 0.000000000484; 0.00000000000172; and 0.00000000006. **0.000000000484; 0.00000000006; 0.00000000000172**

- Ask students to identify what made these exercises tedious. **Possible answers: Having to rewrite the numbers to compare like place values; having to count many zeros to compare.**

- Tell students that many measurements in scientific applications involve numbers that are very large or very small, like those in the exercises. Ask students to give examples of such measurements. **Possible answers: Distances in space, diameter of atom, national debt**

- Point out to students that working with numbers that contain many digits may be cumbersome. It may be difficult to compare, to order, and to perform arithmetic operations on them. For this reason, a shorthand notation for these numbers is used. Students will learn this new notation (scientific notation) in this lesson.

Subtopic 1 — Powers of Ten with Integer Exponents

Expand Their Horizons

This subtopic shows students how to convert between the exponential and standard forms of powers of 10. In standard form, a power of 10 with an integer exponent is a number that has only the digit zero or one in any given place. The powers of 10 with integer exponents are the basis for our decimal number system.

As students work with numbers that have many digits, encourage them to use good work habits. Remind them that numbers greater than 9999 must be written using commas where appropriate. Some students may even benefit from using slight spacing between periods of decimal place values. For example, they can write 0.000 000 000 007.

1 We know that 1,000,000 is a power of 10 with an integer exponent because it uses the digit zero or one in each place value. To write in exponential form, count the number of zeros. Use the number of zeros as the power. Since 1,000,000 has six zeros, $1,000,000 = 10^6$.

2 The number 0.00001 is a power of 10 less than one. The digit one lies in the fifth decimal place. $0.00001 = 10^{-5}$.

3 To evaluate 10^7, write the digit one followed by seven zeros. $10^7 = 10,000,000$. Point out to students that the study of scientific notation makes it simpler to write the expression 10^7 than the number 10,000,000.

5.7B

4 To evaluate 10^{-7}, write a decimal point. The digit one lies in the seventh decimal place to the right of the decimal point. $10^{-7} = 0.0000001$.

Common Error Alert:

Students often confuse the rules for powers of 10 greater and less than one. They may use seven zeros in the decimal part of the number to write 10^{-7} in standard form, writing 0.00000001 instead of 0.0000001. Remind them that for negative integer powers of 10 the opposite of the exponent tells how many decimal places there are in the number.

Additional Examples

1. **Write as a power of 10.**
   100,000,000   0.00000001

   For 100,000,000, count the number of zeros. For 0.00000001, count the number of decimal places.

   $$100,000,000 = 10^8$$
   $$0.00000001 = 10^{-8}$$

2. **Evaluate the expression.**
   10^{-6} 10^6

   For 10^{-6}, write a decimal point followed by six decimal places. Write the digit one in the sixth decimal place and precede it with zeros. For 10^6, write the digit one followed by six zeros.

   $$10^{-6} = 0.000001$$
   $$10^6 = 1,000,000$$

Subtopic 2 ▼ Multiply by a Power of Ten with an Integer Exponent

Expand Their Horizons

In this subtopic, students find the product of a number and a power of 10 with an integer exponent. This skill helps lay the foundation for the study of scientific notation, which will be studied in the next subtopic.

This subtopic provides students with written rules for multiplying a number by a power of 10. To find the product, the decimal point in the number is moved either to the left or right a number of places determined by the power of 10. Students sometimes confuse the rules and move the decimal point in the wrong direction. When multiplying a positive number by a positive power of 10, the product will be greater than the number; when multiplying a positive number by a negative power of 10, the product will be less than the number. While going through the examples and exercises for this subtopic, ask students to make observations about the relative size of the products and to formulate their own understanding of what is happening as the powers of 10 go from positive to negative exponents and the effect on the placement of the decimal points.

5.7C

Remind students that positive numbers are often associated with the right-hand direction. For example, positive numbers lie to the right of zero on the number line. One way to remember which direction to move the decimal point is to think *positive, right; negative, left*.

Writing numbers in expanded form is a useful exercise in preparation for the study of scientific notation because it reminds students that there are many ways of writing a given number. For example, the number 845 could be written in standard form (845), or in an expanded form (such as eight hundreds, four tens, five ones; or 8 x 100 + 4 x 10 + 5 x 1; or 800 + 40 + 5) or in word form (eight hundred forty-five). Tell students that each form is valid, but that certain forms may be more useful in certain problem situations than others.

5 The expression shows a number multiplied by a positive power of 10. To find the product, move the decimal point in the first factor three places to the right.
$14.25 \times 10^3 = 14{,}250$

6 The expression shows 0.35 multiplied by a negative power of 10. To find the product, move the decimal point in 0.35 one place to the left. $0.35 \times 10^{-1} = 0.035$

7 To write in expanded form, write the number as the sum of the values of each digit, found by multiplying the digit by the place value.
$4.075 = 4 \times 10^0 + 0 \times 10^{-1} + 7 \times 10^{-2} + 5 \times 10^{-3}$
Students may understand expanded form better if parentheses are used to group the product of each digit and its value.
$4.075 = (4 \times 10^0) + (0 \times 10^{-1}) + (7 \times 10^{-2}) + (5 \times 10^{-3})$ or $4.075 = 4 + 0.07 + 0.005$

Additional Examples

1. **Multiply.**
$$3.2 \times 10^6$$

Move the decimal point in 3.2 six places to the right.

$$3.2 \times 10^6$$
$$3.\underset{1\ 2\ 3\ 4\ 5\ 6}{\overrightarrow{2}\overrightarrow{0}\overrightarrow{0}\overrightarrow{0}\overrightarrow{0}\overrightarrow{0}}.$$
$$3{,}200{,}000$$

2. **Write in expanded form.**
$$25.3007$$

Write 25.3007 as the sum of the values of its digits.

$$(2 \times 10^1) + (5 \times 10^0) + (3 \times 10^{-1}) + (7 \times 10^{-4})$$

or

$$20 + 5 + 0.3 + 0.0007$$

Part Two

Instructional Alert

To access the second portion of the lesson on the DVD menu, select "Part Two" on the 5.7 Lesson Menu.

5.7D

Expand Their Horizons

In this subtopic, students are introduced to scientific notation. They learn how to recognize whether or not a given number is written correctly in scientific notation.

Take a moment to discuss the term *notation* with the class. A notation is a *way of writing*. Standard notation is the usual way of writing numbers using digits and place value. Scientific notation is simply another way of writing numbers, and it is generally most useful when writing numbers that are very large or very small. Point out that any real number can be written using this notation.

8 The expression shows the product of 5.6 and 10^7. Since 5.6 is at least one, but less than 10, and because 10^7 is an integer power of 10, 5.6×10^7 is a number written in scientific notation.

9 Since 0.4 is less than one, 0.4×10^{-5} is not written in scientific notation.

10 Because eight is at least one and less than 10, and because 10^{87} is an integer power of 10, 8×10^{87} is written in scientific notation.

Additional Examples

1. **Is this in scientific notation?**
   $$0.51 \times 10^{-4}$$

   The first factor must be at least one but less than 10. The second factor must be an integer power of 10.

   Since 0.51 is less than one, the number is *not* in scientific notation.
   NO

2. **Is this in scientific notation?**
   $$3.2 \times 10^{12}$$

   The first factor must be at least one but less than 10. The second factor must be an integer power of 10.

   Both factors have the correct form. The number *is* in scientific notation.
   YES

5.7E

Expand Their Horizons

In this subtopic, students will convert between standard and scientific notations. The subtopic begins by converting numbers greater than one. Be sure to point out to the students that each power of 10 in the second factor will be positive. Then the students will be converting numbers less than one. Point out that when numbers less than one are written in scientific notation, the power of 10 is less than zero. (That is, the power of 10 is negative). Each time they convert a number from one form to the other, encourage them to check their work by converting back to the other form.

11 Place the new decimal point after the eight. Count the number of decimal places to the original decimal point. $876,000 = 8.76 \times 10^5$

12 To write 6.12×10^{-4} in standard notation, move the decimal point in 6.12 four places to the left. Remind students that they must add zeros for placeholders so they can move four places. $6.12 \times 10^{-4} = .000612$

13 Place the new decimal point after the two. Count the number of decimal places to the original decimal point. $0.00000024 = 2.4 \times 10^{-7}$

14 Move the decimal point in 4.5 two places to the left. $4.5 \times 10^{-2} = 0.045$

Additional Examples

1. Write in scientific notation.
$$32,000,000$$

Place the decimal point after the first nonzero digit. Count the number of decimal places to the original decimal point.

$$32,000,000$$
$$3.\underset{\underset{1\ 2\ 3\ 4\ 5\ 6\ 7}{\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow}}{2000000}.$$
$$3.2 \times 10^7$$

2. Write in standard notation.
$$4.51 \times 10^5$$

Move the decimal point in 4.51 five places to the right.

$$4.51 \times 10^5$$
$$4.\underset{\underset{1\ 2\ 3\ 4\ 5}{\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow}}{51000}.$$
$$451,000$$

3. Write in scientific notation.
$$0.00000051$$

Place the decimal point after the first nonzero digit. Count the number of decimal places to the original decimal point.

$$0.00000051$$
$$0.\underset{\underset{-7\ -6\ -5\ -4\ -3\ -2\ -1}{\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow}}{0000005}.1$$
$$5.1 \times 10^{-7}$$

4. Write in standard notation.
$$8.3 \times 10^{-9}$$

Move the decimal point in 8.3 nine places to the left.

$$8.3 \times 10^{-9}$$
$$0.\underset{\underset{-9\ -8\ -7\ -6\ -5\ -4\ -3\ -2\ -1}{\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow}}{00000000}8.3$$
$$0.0000000083$$

This lesson teaches students to read and write numbers written in scientific notation. In future courses, they will use numbers written in this form to perform operations. To challenge students, show them the expression $(4.5 \times 10^3) \times (2 \times 10^5)$ and ask them to evaluate it. Remind them that they can use the Associative and Commutative Properties to make the problem $(4.5 \times 2) \times (10^3 \times 10^5)$. Then, they can use multiplication and what they know about exponents to simplify the expression to 9×10^8.

Calculator

Scientific and graphing calculators can be used to convert numbers between scientific and standard notations. These calculators generally offer the user a choice in how numbers are displayed on the screen. On a Texas Instruments graphing calculator, "normal" mode displays numbers using standard notation. In "sci" (scientific notation) mode, numbers are displayed in scientific notation using shorthand notation in which the first factor is followed by an E, which is followed by the integer power of 10.

To convert a number from scientific notation to standard form, set the calculator to normal mode. To enter the number, enter the first factor, press the EE key and then enter the power of 10. Press the ENTER key. The number will appear in standard notation.

To convert a number from standard to scientific notation, set the calculator to scientific notation mode. Enter a number and then press the enter key. The number will appear in scientific notation.

In normal mode:

```
5ε2
               500
4.65ε7
          46500000
6.2ε1
                62
■
```

In scientific notation mode:

```
6500000
             6.5ε6
0.000000051
            5.1ε-8
15
             1.5ε1
```