Numbers and Operations

Module 5

Decimal Operations, Exponents, and Powers

Lesson 6
Exponents and Powers

Teacher Notes 5.6

Objectives

- Use factors of numbers to introduce exponents and powers.
- Demonstrate an understanding of exponents and powers and an understanding of when to use exponents and powers in expressions.
- Define and use negative exponents.
- Solve problems with exponents and powers.

Prerequisites

Adding, subtracting, multiplying, and dividing integers

Reading and writing fractions

Using the Order of Operations

Vocabulary

Exponent (1.1)
Base (1.1)
Factor (1.2)
Square of a number (1.1)
Expression (1.1)
Cube of a number (1.1)
Power (1.1)
Exponential form of an expression
Even number (1.2)
Odd number (2.5)
Order of operations (1.1)
Whole number (3.1)
Negative exponent
Nonzero number
Evaluate (1.1)
Zero exponent

Get Started

- The *Operations Face-off* game will help students review the Order of Operations needed to evaluate expressions with exponents and powers:
- Draw lines on a piece of paper to divide it into 13 rows and four columns. In each of the resulting 52 sections, write an expression that contains at least two operations. Include exponents in some of the expressions. Possible expressions are $2 + 5 \times 7$; $9 \div (5 - 2)$; 6^2; $10 - 2^2$; and $(3 + 1) \times (8 - 2)$.
- Make enough copies so that each pair of students gets one copy. Provide scissors for students to cut the paper into 52 "cards." Have each pair of students place their cards in a paper bag.
- Quickly review the Order of Operations. Review the acronym PEMDAS and discuss its meaning. Operations are carried out in the order **P**arentheses, **E**xponents, **M**ultiplication/**D**ivision, and **A**ddition/**S**ubtraction.

- To play *Operations Face-off*, each student pulls a card from the bag and evaluates the expression. The player who drew the card holding the expression of greater value wins the round and collects the two cards.
- If the expressions have the same value, another pair of cards is drawn until the expressions have different values. The player with the card holding the expression of greater value wins all the cards. Play ends when all cards have been used. The player with the most cards at the end of the game wins.
- Tell the class that today's lesson will expand upon what they already know about evaluating expressions using the Order of Operations. Today they will study expressions containing negative and zero exponents.

Subtopic 1 — Exponents and Powers

Expand Their Horizons

This subtopic provides a review of exponents, including the vocabulary *base*, *exponent*, and *power*. Before viewing the lesson, remind students that an exponent indicates repeated multiplication.

Common Error Alert:

Some students may misinterpret the operation indicated by a power. For example, they may interpret 2^3 as meaning 2 × 3. Warn them that this mistake is common and suggest they be extra diligent in checking their work in order to avoid using repeated addition instead of repeated multiplication.

When evaluating the expression -5^4, show students that the negative sign can be placed outside the parentheses. Ex. $-(5^4) = -625$ or the opposite of 625.

Remind students that using the properties of real numbers to evaluate exponents can make the job simpler. For example, when evaluating 2^6, or 2 × 2 × 2 × 2 × 2 × 2, students can use the Associative Property to group the factors rather than simply multiplying the factors from left to right.

$$2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$(2 \times 2) \times (2 \times 2) \times (2 \times 2)$$
$$4 \times 4 \times 4$$
$$64$$

1 The expression is shown in expanded form. It shows negative six used as a factor five times. $(-6) \times (-6) \times (-6) \times (-6) \times (-6) = (-6)^5$

2 Remind students that to evaluate an expression is to complete all indicated operations. To evaluate 7^3, use seven as a factor three times.
$7^3 = 7 \times 7 \times 7 = 343$

3 This expression is given in word form. The exponential expression is 2^6. In expanded form, the expression is 2 × 2 × 2 × 2 × 2 × 2, or 64.

Additional Examples

1. Evaluate.

$$3^4$$

Use three as a factor four times.

$$3^4$$
$$3 \times 3 \times 3 \times 3$$
$$81$$

2. Evaluate.

$$-2^3$$

Use two as a factor three times. Take the opposite of the product.

$$-2^3$$
$$-(2 \times 2 \times 2)$$
$$-8$$

Subtopic 2 Using Exponents and Powers in Expressions

Expand Their Horizons

In this subtopic, students evaluate expressions containing exponents and powers. If the Get Started activity were used, students will have already reviewed Order of Operations. If not, take a moment to review the topic before viewing the DVD.

If students are having trouble carrying out operations in the correct order, suggest that they develop a plan before starting. Ask them to list the operations in the order in which they will be carried out. Students should wait for approval before proceeding.

4 In each factor, parentheses indicate that the negative sign is to be included in the base. The order of operations for this expression is exponents and then multiplication.

$$(-4)^2 \times (-3)^3$$
$$(-4 \times -4) \times (-3 \times -3 \times -3)$$
$$16 \times -27$$
$$-432$$

5 Simplify inside parentheses, evaluate the exponent, and then multiply.

$$2(9 - 6)^2$$
$$2(3)^2$$
$$2(9)$$
$$18$$

Common Error Alert:

When evaluating expressions containing several operations, students sometimes become impatient or careless. Encourage them to work carefully, performing one operation at a time and rewriting the expression on a new line after each step.

6 Evaluate exponents and then subtract.

$$3^3 - 2^3$$
$$(3 \times 3 \times 3) - (2 \times 2 \times 2)$$
$$27 - 8$$
$$19$$

Common Error Alert:

When simplifying expressions like $3^3 - 2^3$, students sometimes see like exponents and assume that they can subtract the bases since the exponents are the same. So, they might write $3^3 - 2^3 = 1^3$. Remind them to double-check their work to be sure they followed the correct order of operations.

Additional Examples

1. Evaluate.

$$\mathbf{(4 - 7)^2}$$

Subtract and square the result.

$$(4 - 7)^2$$
$$(-3)^2$$
$$-3 \times -3$$
$$9$$

2. Evaluate.

$$\mathbf{4^3 \div (5 - 3)^2}$$

Subtract, evaluate exponents, and then divide.

$$4^3 \div (5 - 3)^2$$
$$4^3 \div (2)^2$$
$$(4 \times 4 \times 4) \div (2 \times 2)$$
$$64 \div 4$$
$$16$$

Expand Their Horizons

In this subtopic, students are introduced to zero and negative exponents.

Students are sometimes perplexed by negative exponents. When they evaluate a negative exponent, they may expect a negative number as a result, and this is not always the case. Tell them to think of a negative exponent as a set of instructions: make a fraction with a numerator of one, use the base and the opposite of the negative exponent as the denominator, and then simplify.

For students who ask why negative exponents are evaluated this way, ask them to consider the following equations:

$$2^4 = 16$$
$$2^3 = 8$$
$$2^2 = 4$$
$$2^1 = 2$$

Ask students to observe and to describe the pattern for both sides of the equation. **The number on the right side is the quotient above it divided by two. So, to get each line of the pattern, decrease the exponent by one, and divide the number on the right side by two.** Ask the students to continue the pattern to develop the resulting meaning of a negative exponent.

$$2^0 = 2 \div 2 = 1$$
$$2^{-1} = 1 \div 2 = \tfrac{1}{2}$$
$$2^{-2} = \tfrac{1}{2} \div 2 = \tfrac{1}{4}$$

Some students may observe the use of the word “nonzero” in the definitions for zero and negative exponents. Tell them the expression 0^0 is undefined. A number cannot be divided into zero groups or divided into groups with zero items. Because division by zero is undefined, zero raised to a negative power is also undefined.

7 Use the definition of a negative exponent and evaluate the power in the denominator.

$$3^{-4}$$
$$\frac{1}{3^4}$$
$$\frac{1}{3 \times 3 \times 3 \times 3}$$
$$\frac{1}{9 \times 9}$$
$$\frac{1}{81}$$

8 Evaluate exponents and then multiply.

$$2^6 \times 8^0$$
$$64 \times 8^0$$
$$64 \times 1$$
$$64$$

Additional Examples

1. Evaluate.

$$(3^3 - 4)^0$$

Any nonzero quantity raised to the zero power is one.

$$(3^3 - 4)^0$$
$$(27 - 4)^0$$
$$(23)^0$$
$$1$$

2. Evaluate.

$$2^{-3} \times 5^0$$

Evaluate exponents. Then multiply.

$$2^{-3} \times 5^0$$
$$\frac{1}{2^3} \times 5^0$$
$$\frac{1}{8} \times 1$$
$$\frac{1}{8}$$

Subtopic 4 Solving Problems with Exponents and Powers

Expand Their Horizons

In the previous subtopic, students solved problems using exponents. The problems in this subtopic could be solved without the use of exponents. However, encourage students to write and use exponential expressions to model each problem.

9 Students might approach this problem in various ways. On the DVD, each number is written in expanded form, showing two used as a factor 13 times. So, the product is 2^{13}, or 8,192. Some students might solve by evaluating 2^3 and 2^{10}, then multiplying: $2^3 \times 2^{10} = 8 \times 1{,}024 = 8{,}192$. The process used on the DVD hints at a rule for multiplying powers that have the same base. In the Challenge Problems, students are asked to make a conjecture for this rule.

10 Point out to English Language Learners that to *double* an amount is to multiply it by two. To solve, write expressions for the amount received each day. Use the expression for the previous day's amount to write each day's amount. The amount for Day 5 is $\$4 \times 2 \times 2 \times 2 \times 2$, or \$64.

Additional Examples

1. A password must contain four characters. The first character must be a lower-case letter, and the others must be numbers. How many passwords are possible?

Model the four places of the password. Determine the number of possible characters for each place. Multiply to find the number of possible passwords.

□	□	□	□
letter	number	number	number
26	10	10	10

$26 \times 10 \times 10 \times 10$

26×10^3

$26 \times 1{,}000$

$26{,}000$

There are 26,000 possible passwords.

2. A zoo's collection of birds tripled every year. In the first year, there were four birds. How many birds were there in the fifth year?

Write expressions to show the number of birds in each year. Simplify the expression for Year 5.

Year 1: 4
Year 2: 4×3
Year 3: $4 \times 3 \times 3$
Year 4: $4 \times 3 \times 3 \times 3$
Year 5: $4 \times 3 \times 3 \times 3 \times 3$

4×3^4

4×81

324

There were 324 birds in the fifth year.

Look Beyond

This lesson presented students with expressions of the form $a \times b^x$, where a and b are nonzero numbers and x is an integer. In the next lesson, students will study scientific notation, in which numbers are written as the product of a certain nonzero number and an integer power of 10. Familiarity with zero and negative exponents is crucial to understanding scientific notation.

In algebra, students will study exponential functions of the form $y = ab^x$. In particular, functions that increase by the same percent in a given time period are called exponential growth functions. Functions that decrease by the same percent in a given time period are called exponential decay functions.

Connections

Powers can be used to find the total number of license plates that can be issued for a state. For example, one state may have license plates that are made up of two letters and four numbers. There would be a total of $26^2 \times 10^4$ different license plate numbers. A state that uses three letters and three numbers can issue $26^3 \times 10^3$ different license plate numbers.

Calculator

Students might be interested in seeing how zero and negative exponents can be evaluated on a calculator. Most scientific calculators have a key for evaluating exponents (for example, many Texas Instruments calculators have the key [y^x]). To evaluate a power, enter the base, press the [y^x] key, and then enter the exponent. If the exponent is negative, the [+/-] key can be used to change the sign. To evaluate 3^{-4}, press the following series of keys:

[3] [y^x] [4] [+/-] [=]

The answer will appear in decimal form.