Numbers and Operations

★ Module 5 ★

Decimal Operations, Exponents, and Powers

Lesson 5
Dividing Decimals

Teacher Notes 5.5

Objectives

- Estimate quotients using rounding and compatible numbers.
- Model division of decimals using diagrams and/or illustrations of manipulatives.
- Develop and use algorithms to divide decimals (hundredths by tenths up to thousandths by thousandths).

Vocabulary

Estimate (1.5)
Rounding (1.5)
Quotient (2.4)
Decimal (4.2)
Dividend (2.4)
Divisor (2.4)
Compatible numbers (1.5)
Place value (2.1)
Repeating decimal
Terminating decimal

Prerequisites

Rounding whole numbers and decimal numbers

Dividing multi-digit whole numbers

Using place value of whole numbers and decimals

Writing fractions as decimals

Estimating quotients of whole numbers

Instructional Alert

In order to adequately cover the content in this lesson, the lesson has been divided into two parts in an appropriate and logical place. Each part of the lesson may be presented in separate class periods or on separate days. In schools with extended class periods, teachers may present both parts as one lesson.

Part One

Get Started

- Divide the class into two groups: a dividend group and a divisor group.
- Give each student an index card. Have the dividend group write any whole number between 100 and 1,000 on their card. Have the divisor group write any whole number between 10 and 50 on their card.
- Randomly choose one student from each group. Tape their numbers to the board and write a division sign between them.
- Tell students to estimate the quotient. Remind them to use compatible numbers.
- Have two or three students share their estimates with the class. Discuss which compatible numbers were chosen and explain why they were chosen.
- Repeat with several more pairs of students.

Subtopic 1 — Estimating Quotients Using Front-end Estimation, Rounding, and Compatible Numbers

Expand Their Horizons

In this subtopic, students estimate the quotient of two decimal numbers. They first learn that multiplying the dividend and divisor by the same number does not change the quotient. With that, they rewrite decimal division problems so that the divisor is a whole number. Then they divide compatible numbers.

Stress that the goal is to multiply by a power of 10 that makes the divisor, not necessarily the dividend, a whole number. It is okay for the dividend to still have a decimal point. To estimate 616.25 ÷ 30.2, multiply both numbers by 10, which moves each decimal point one place to the right: 6,162.5 ÷ 302. Then find compatible numbers: 6,000 ÷ 300 = 20. The estimate of 616.25 ÷ 30.2 is 20.

1 Because the divisor has one decimal place, multiply both numbers by 10. Because the divisor is an even number, round the dividend to an even number and divide.

There may be more than one correct estimate to a problem. The dividend 403 could have been rounded to 404 for an estimate of 202. Both estimates are close to the exact answer of 201.5.

2 Multiply both numbers by 100. Use the basic fact $5 \times 11 = 55$ to find the most compatible number for 5,429, which is 5,500.

Additional Examples

1. Estimate.

378.5 ÷ 1.9
Multiply both factors by 10.

(378.5 × 10) ÷ (1.9 × 10) = 3,785 ÷ 19

Find compatible numbers.

3800 ÷ 19 = 200

378.5 ÷ 1.9 ≈ 200

2. Estimate.

6.4 ÷ 0.82
Multiply both factors by 100.

(6.4 × 100) ÷ (0.82 × 100) = 640 ÷ 82

Find compatible numbers.

640 ÷ 80 = 8

6.4 ÷ 0.82 ≈ 8

Subtopic 2 — Dividing Decimals Using Models

Expand Their Horizons

In this subtopic, students model division problems which involve money. They use patterns, a hundredths-square, and diagrams.

First, students discover a pattern in dividing by decreasing powers of 10. The decimal point in the quotient moves one place to the left every time the power of 10 decreases. Next they discover that a quotient is not always smaller than the dividend. This concept can be tricky for some students, because they tend to think that the dividend is always larger than either the divisor or quotient. For instance, $10 divided by two people is $5 each. The number 10 is larger than the two and five. However, when finding how many nickels are in $10, the problem is $10 ÷ $0.05, and the quotient is 200, which is larger than both 10 and 0.05.

Teachers may wish to describe a situation that does not involve money. Tell students that a certain clock chimes every half hour. Ask how many times it will chime in three hours. They should answer six times. Show that this situation can be written as 3 ÷ 0.5, and that their quotient was greater than both the dividend and divisor. Students should be reminded of the connection between multiplication and division. In the previous module on multiplication of decimals, a product can be smaller than both the factors and any multiplication problem can be turned into a related division problem. Ex. 6 x 2 = 12, so 12 ÷ 2 = 6, and 6 x 0.02 = 0.12, so 0.12 ÷ 0.02 = 6.

Students are also shown that money problems can be modeled by drawing and separating coins into groups and by shading in a hundredths-square, where a dime is a column and a nickel is five blocks, or half a column.

Common Error Alert:

When finding the decimal part of a quotient, students may simply write the remaining number of coins as the decimal part. For example, when finding the number of quarters in $1.40, students may write 5.3 because there are three remaining nickels. Remind them that their answer must be written as part of a quarter; a quarter is five nickels, so they have to find the decimal for $\frac{3}{5}$, which is 0.6. As a result, the correct answer is 5.6 quarters.

3 Divide each amount by two. Each dividend is a decreasing power of 10, so each subsequent quotient is formed by moving the decimal point one place to the left.

4 Model $1.30 using coins. Because students are dividing by quarters, use as many quarters in the model as possible. The one remaining nickel is one fifth of a quarter, which is equivalent to 0.2.

For the sake of simplicity, one nickel, instead of five pennies, is used for the remaining five cents. Notice, however, five pennies is still one fifth of a quarter because five twenty fifths reduces to one fifth.

Additional Examples

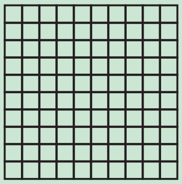
1. If each amount is shared equally by five people, how much would each person get?

$3,000
$300
$30
$3
$0.30

Notice that each amount is a decreasing power of 10. Find the first quotient, and for each subsequent problem, move the decimal point one place to the left.

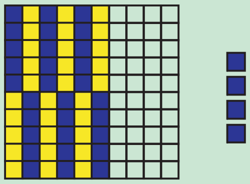
$3,000 ÷ 5 = $600
$300 ÷ 5 = $60
$30 ÷ 5 = $6
$3 ÷ 5 = $0.60
$0.30 ÷ 5 = $0.06

2. How many nickels are in 0.64? What, if anything, is left over?



Model 0.64. There are 12 half columns, which represent 12 nickels. Write the remaining four cents as four fifths of a nickel. Four fifths is equal to eight tens or 0.8.

12.8 nickels



Part Two

Instructional Alert

To access the second portion of the lesson on the DVD menu, select “Part Two” on the 5.5 Lesson Menu.

Subtopic 3 — Dividing Decimals by Whole Numbers

Expand Their Horizons

In this subtopic, students first learn that the decimal point in the quotient is directly above the decimal point in the dividend. Next, they divide. The division algorithm is the same as with whole numbers divided by whole numbers. Students learn how to add zeros as placeholders and how to recognize and write repeating decimals.

Common Error Alert:

Students may forget to add zeros to the dividend to complete the division problem. For example, when dividing seven by two, they may not add the zero to the right of the seven. This mistake may lead them to write the difference, one, as part of the quotient.

$$\begin{array}{r} 3 \\ 2\overline{)7} \\ \underline{-6} \\ 1 \end{array} = 3.1 \qquad \text{instead of} \qquad \begin{array}{r} 3.5 \\ 2\overline{)7.0} \\ \underline{-6} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

Remind students that they want the final subtraction to end in zero. If this does not happen, they should look for a group of digits that repeat.

5 This quotient is a repeating decimal. Notice that the last difference shown is the same as the original dividend. Therefore, continuing the process will result in the same differences over and over.

6 Divide until the difference is zero. No extra zeros are needed in either the quotient or dividend.

Additional Examples

1. Estimate and divide.

8 ÷ 9

Find compatible numbers and divide.
8 ÷ 10 = 0.8

$$\begin{array}{r} .8 \\ 9\overline{)\,8.0} \\ \underline{-72} \\ 8 \end{array}$$

The difference is the same as the dividend. The pattern will repeat.

$8 \div 9 = 0.\overline{8}$

2. Estimate and divide.

193.2 ÷ 28

Find compatible numbers and divide.

210 ÷ 30 = 7

$$\begin{array}{r} 6.9 \\ 28\overline{)193.2} \\ \underline{-168} \\ 252 \\ \underline{-252} \\ 0 \end{array}$$

Subtopic 4 — Dividing Decimals by Decimals

Expand Their Horizons

In this subtopic, students find quotients when the divisor is a decimal. They are reminded of the problems in Subtopic 1, when they estimated quotients by multiplying both the dividend and divisor by a power of 10.

Common Error Alert:

Students may move the decimal point in the divisor but may not move it in the dividend. Remind students that the process only works when both numbers are multiplied by the same amount. Encourage them to start each problem with this three-step process: move the decimal in the divisor, move the decimal in the dividend, and place the decimal in the quotient.

7 Move the decimal point in each number one place to the right and divide.

8 The divisor has three decimal places. When moving the decimal point in the dividend, add a zero as a placeholder.

Additional Examples

1. Estimate and divide.

144 ÷ 4.8

Estimate: 150 ÷ 5 = 30

Move each decimal point one place to the right.

$$4.8\overline{)144} = 48\overline{)1{,}440}$$

$$\begin{array}{r} 30 \\ 48\overline{)1{,}440} \\ -144 \\ \hline 00 \\ -00 \\ \hline 0 \end{array}$$

2. Estimate and divide.

9.45 ÷ 3.5

Estimate: 90 ÷ 30 = 3

Move each decimal point one place to the right.

$$3.5\overline{)9.45} = 35\overline{)94.5}$$

$$\begin{array}{r} 2.7 \\ 35\overline{)94.5} \\ -70 \\ \hline 245 \\ -245 \\ \hline 0 \end{array}$$

Look Beyond

Students will multiply and will divide decimals by powers of 10 when writing very large or very small numbers in scientific notation. This means they will be moving the decimal point in a number either to the right or to the left, depending on the value of the number.

Scientific notation is named as such because very large and very small numbers appear often in the sciences: very large numbers are needed for distances in space; very small numbers are needed for lengths of bacteria and viruses.

Connections

Decimal division can be used at the supermarket to find unit prices for different sizes and brands of items. For example, if 10.5 ounces of one brand of soup costs $1.45, consumers can use compatible numbers and can estimate that each ounce costs about $0.15. If another soup has a unit price of $0.12, consumers may wish to purchase that soup instead. Three cents per ounce may seem like a small savings, but if buying a large amount for a party, it could mean the difference between staying within and going over a budget.