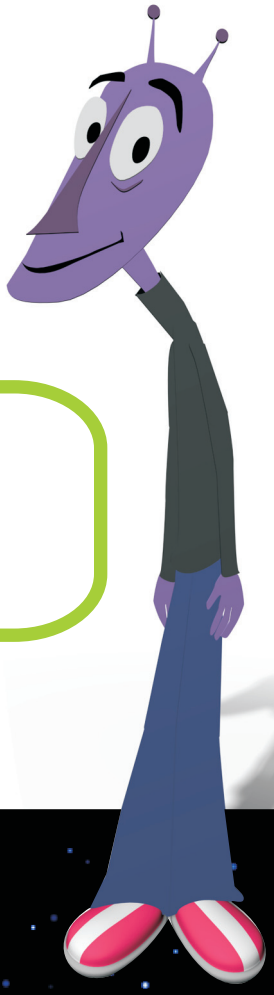
# Numbers and Operations

## ★ Module 5 ★

## Decimal Operations, Exponents, and Powers

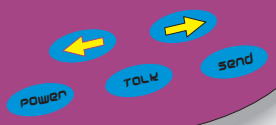


### Lesson 4
### Multiplying Decimals

# Teacher Notes 5.4

**Objectives**

- Estimate products using rounding and compatible numbers.
- Model multiplication of decimals using diagrams and/or illustrations of manipulatives.
- Develop and use algorithms to multiply decimals (hundredths × tenths up to thousandths × thousandths).




## Prerequisites

Multiplying multi-digit whole numbers

Rounding whole numbers and decimals

Using place value of whole numbers and decimals

Estimating products of whole numbers

## Vocabulary

Decimal (4.2)
Commutative Property of Multiplication (1.3)
Product (1.1)
Factor (1.2)
Estimate (1.5)
Rounding (1.5)
Place value (2.1)
Placeholder
Array (4.3)
Hundredths-square (4.1)

## Instructional Alert

**In order to adequately cover the content in this lesson, the lesson has been divided into two parts in an appropriate and logical place. Each part of the lesson may be presented in separate class periods or on separate days. In schools with extended class periods, teachers may present both parts as one lesson.**

## *Part One*

### Get Started

- Tell students, “I have five pennies.” Have a volunteer write the amount of money, as a decimal, on the board. **$0.05**
- Now say, “To be able to buy a pen, I need ten times as much money as that. How much money do I need?” **Fifty cents** Have another volunteer write this amount on the board. **$0.50**
- Tell students that a lunchbox is ten times as expensive as a pen. Have another volunteer write this amount, as a decimal, on the board. If needed, model the amount by drawing 10 sets of “fifty cents” and counting the money. **$5.00**
- Have all students write down, in their notebooks, the two multiplication problems that were just solved.

  **$0.05 × 10 = $0.50**
  **$0.50 × 10 = $5.00**

- Ask students to identify the similarities and differences in the two answers. Elicit that both answers have a five, but the decimal point changes the value of the money. Tell students they will learn that multiplying by 10 just moves the decimal point in the other factor, and multiplying by 100, by 1,000, and by other powers of 10 is just as easy.

## Subtopic 1 Model Multiplication of Decimals

### Expand Their Horizons

In this subtopic, students use models to multiply decimals. When one factor is a decimal and the other is a whole number, they use the fact that multiplication is repeated addition. For the product $0.2 \times 4$, model the number of whole units and then shade the decimal part of each whole. For the product $4 \times 0.2$, shade the decimal number as many times as the whole number. The models show the same two factors multiplied in different orders. It is important for students to see that, in both ways, the same number of blocks is shaded, i.e., the product in both cases is the same, 0.8. This illustrates the Commutative Property of Multiplication.

Next, the products $3 \times 0.4 = 1.2$ and $3 \times 0.04 = 0.12$ are shown. Have students recall the basic fact $3 \times 4 = 12$ and then see if they can predict the product $3 \times 0.004 = 0.012$.

Students then learn how to model products when both factors are decimals. The model used is an array model on a hundredths-square. The product $0.3 \times 0.4$ shows 0.3 groups of 0.4. To do this, the factor 0.4 is modeled vertically and the factor 0.3 is modeled horizontally. The intersection of the two models is the product 0.12. The decimal numbers represent the width and the length of the intersection. This example shows that the product of two positive decimal numbers less than one is less than either of the products. This is a difficult concept for many students. It might help students to be reminded that the decimal, 0.1, is the same as the fraction 1/10. Therefore, $0.1 \times 0.6$ is the same as 1/10 of 0.6.

1

The product is modeled by first shading in six tenths of a hundredths-square. In the opposite direction, five tenths, or one-half of the same hundredths-square is shaded in a different color. The product is the intersection of the two shadings which is 30 blocks, written as 0.30 or 0.3.

Students may be more comfortable seeing 0.3 shaded as three rows or three columns. Show them that the 30 blocks can be rearranged into rows or columns.

Problems 2 and 3 of the Lesson Notes stress the importance of place value. Both problems have a factor of six, but Problem 2 has 0.5 as the second factor and Problem 3 has 0.05 as the second factor. Point out the difference in the products: the first product is three full hundredths-squares; the second product is less than half of a hundredths-square.

2

Five tenths, or one-half, is modeled six times. This requires shading three hundredths-squares, so the product is three.

3

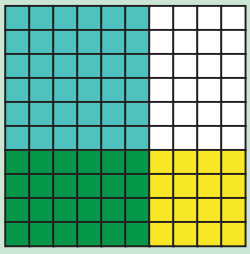
Five hundredths is shaded in a hundredths-square six times. The shading can be done in any direction; the result will be 30 shaded blocks.

## Additional Examples

**1. Use a model to solve.**

**0.4 × 0.6**

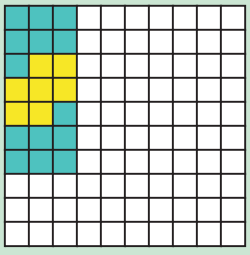
Shade six tenths of the model in one direction. Shade four tenths of the model in the other direction. Count the number of blocks in the intersection, 24. The product is 0.24.



**2. Use a model to solve.**

**3 × 0.07**

Shade seven hundredths of the model three times. Count the number of shaded blocks. The product is 0.21.



## Expand Their Horizons

In this subtopic, students multiply decimal numbers by powers of ten. Students are shown patterns in order to show that the decimal point moves to the right as many places as there are zeros in the power of ten. Remind students that every number can be written as a decimal. So, $4 \times 100$ can be written as $4.0 \times 100$.

Ask students why it makes sense that the decimal point moves to the right when multiplying by powers of ten instead of the left. Draw out from the discussion that moving left would make the number smaller, when it should be getting larger.

**Common Error Alert:**

**Students may count from the right of the last digit of the factor instead of beginning where the decimal point is located in the number. For example, they may answer $24.63 \times 100$ as 246,300 because they moved two places from the right of the three. If students do this, have them first circle the decimal point in the problem and draw arrows showing the movement of the decimal point.**

**Common Error Alert:**

**Students may stop moving the decimal point after they have reached the last digit of the number even when there are more places to move. For example, when solving $4.25 \times 1{,}000$, they may put the decimal point at the end of the five. Remind them that zeros can be added as placeholders, so $4.25 \times 1{,}000$ is the same as $4.250 \times 1{,}000$.**

**4**

In the first example, the decimal point should move two places to the right of its initial position because there are two zeros in 100.

In the second example, the decimal point moves only one place to the right of its initial position because there is one zero in 10.

In the third example, the decimal point moves three places to the right of its initial position and is placed to the right of the last digit.

Teachers may wish to continue the pattern to review the Identity Property of Multiplication.

$2 \times 1{,}000 = 2{,}000$
$2 \times 100 = 200$
$2 \times 10 = 20$
$2 \times 1 = 2$

There are no zeros in the number one, so the decimal point moves no places to the right. It stays where it is; the factor does not change. Any number multiplied by one is that number.

## Additional Examples

**1. Multiply.**

**1.02 × 1,000**

There are three zeros in 1,000. Move the decimal point three places to the right. This places the decimal point one place past the last digit of the factor. Add a zero as a placeholder.

1.02 × 1,000 = 1,020

**2. Multiply.**

**1.542 × 100**

There are two zeros in 100. Move the decimal point two places to the right.

1.542 × 100 = 154.2

## *Part Two*

### Instructional Alert

To access the second portion of the lesson on the DVD menu, select "Part Two" on the 5.4 Lesson Menu.

# Subtopic 3 Estimating Decimal Products

### Expand Their Horizons

In this subtopic, students estimate decimal products before finding the exact answer. Because it is easy for students to misplace a decimal point, this skill will allow them to know if a product they found using the algorithm is reasonable.

To estimate products, students are taught to round one or both factor(s) and then to multiply. Students may come up with different estimates depending on whether they rounded only one factor or both factors and depending on the place to which they chose to round. Teachers may want to have a class discussion about why the students' estimates varied. Example: In the problem 437 x 2.67, one student may round 437 to 400 and 2.67 to 3 and get an estimate of 1200. Another student may round 437 to 440 and 2.67 to 3 and get an estimate of 1320.

To find where to place the decimal point, in both the estimate and exact answer, count the number of decimal places in both factors. The decimal point is then placed in the product so that it has that total number of decimal places. To place the decimal, start at the end of the number and count left.

**5** The product has five decimal places in it because the first factor has three decimal places and the second factor has two decimal places.

Have students compare the estimate and the exact answer. Discuss what happens if both numbers are rounded down—the exact answer is greater than the estimate. If both numbers are rounded up, the exact answer is less than the estimate.

**6** The product has three decimal places in it because the first factor has one decimal place and the second factor has two decimal places.

Have students compare the estimates and the exact answers. Discuss what happens if both numbers are rounded down—the exact answer is greater than the estimate. If both numbers are rounded up, the exact answer is less than the estimate.

### Additional Examples

**1. Estimate before multiplying.**

**31.09 × 2.2**

31.09 → 30
2.2 → 2

30 × 2 = 60
The estimated product is 60.

$$\begin{array}{r} 31.09 \\ \times\ \ 2.2 \\ \hline 6218 \\ +62180 \\ \hline 68.398 \end{array}$$

The product is 68.398.

**2. Estimate before multiplying.**

**48 × 0.065**

48 → 50
0.065 → 0.07

50 × 0.07 = 3.5
The estimated product is 3.5.

$$\begin{array}{r} 0.065 \\ \times\ \ 48 \\ \hline 520 \\ +\ 2600 \\ \hline 3.120 \end{array}$$

The product is 3.12.

## Subtopic 4 Multiplying Decimals Using the Standard Method

### Expand Their Horizons

The problems in this subtopic are solved using the Standard Decimal Multiplication algorithm. Although they are not first estimating the product, advise students to estimate to check their work.

**Common Error Alert:**

**Students may write the factor with fewer digits as the first factor. They may write $6.7 \times 41.9$ as $\begin{array}{r} 6.7 \\ \times\ 41.9 \end{array}$ instead of $\begin{array}{r} 41.9 \\ \times\ 6.7 \end{array}$ for example.**

**Tell students that setting up decimal multiplication is similar to setting up whole number multiplication. Write the factor with the greater number of digits above the factor with fewer digits.**

In Problem 7 of the Lesson Notes, students complete a word problem which involves money. Because money only goes to the hundredths place, the final answer which has three decimal places is rounded to the hundredths place. Stress the importance of thinking of the situation and checking for the reasonableness of answer before writing the final answer to any word problem.

**7** Multiply the factors and move the decimal point so there are three decimal places in the product. Then round because it is money. Remind students they often need to include units, in this case a dollar sign, when answering word problems.

**8** A student could write two lines of partial sums which would each have several zeros, but it is quicker to use the basic math fact of three times two is six. Then move the decimal point six places to the left, which puts five zeros before the six.

## Additional Examples

**1. Bananas cost $0.28 per pound. Jose bought 3.2 pounds of bananas. How much did Jose pay for the bananas?**

$$\begin{array}{r} \$0.28 \\ \times\ \ 3.2 \\ \hline 56 \\ +840 \\ \hline .896 \end{array}$$

Jose paid $0.90 for the bananas.

**2. Multiply.**

**$0.008 \times 0.02$**

$8 \times 2 = 16$

0.008 has three decimal places.
0.02 has two decimal places.

The product has five decimal places.

0.00016

## Look Beyond

The patterns students saw in Multiplying Decimals by Powers of Ten will extend into the division of decimals. They will divide by powers of 10, but this time, they will move the decimal point to the left instead of to the right. Also, like multiplication, estimating to check final answers will be an essential skill because decimal points will again be moved around.

## Connections

The metric system is based on powers of 10. Knowing how to multiply (and later divide) by powers of 10 makes converting from one unit to another quite simple. It is just a matter of moving the decimal point the correct direction and the correct number of places. For instance, to convert from centimeters to millimeters, multiply by 10: 14 cm = 140 mm. To convert from meters to centimeters, multiply by 100: 14 m = 1,400 cm.