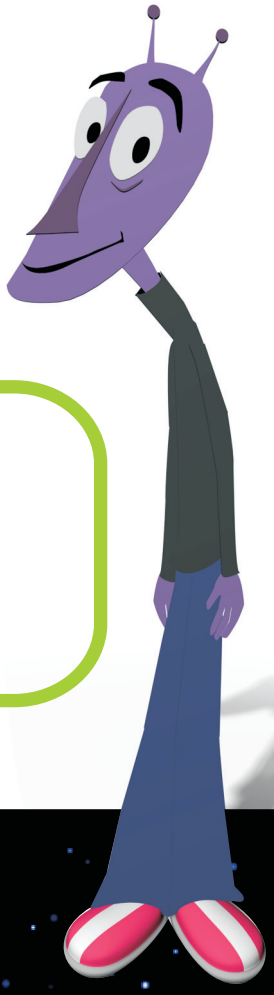
Numbers and Operations

★ Module 5 ★

Decimal Operations, Exponents, and Powers



Lesson 2
Converting, Comparing, and Ordering

Teacher Notes 5.2

Objectives

- Convert, compare, and order fractions, decimals, and percents and find their approximate location on a number line.
- Compare and represent integers, fractions, and decimals and find their approximate location on a number line.

Prerequisites

Reading, writing, and interpreting fractions, decimals, and percents

Modeling fractions, decimals, and percents on a number line

Converting among fraction, decimal, and percent forms of benchmark fractions

Converting between decimals and percents

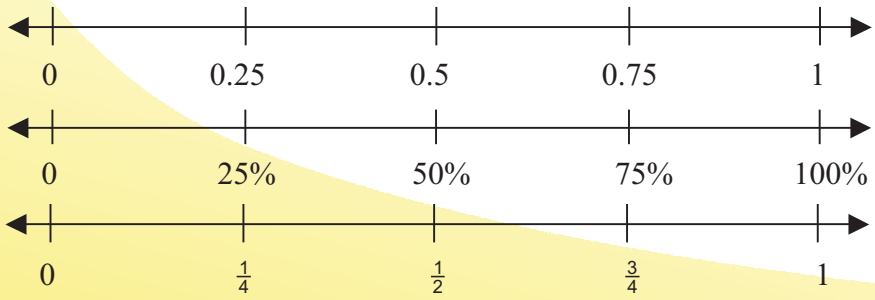
Comparing decimals

Vocabulary

Decimals (4.2)
Place value (2.1)
Equivalent decimal (4.2)
Equivalent percent (4.2)
Benchmark fraction (4.1)
Scale

Get Started

- Prepare a graphic illustration showing the three different number lines shown below on a large piece of poster board, on the chalkboard, or on the overhead projector.



- Point out that each number line shows one whole unit. The top number line uses decimals; the middle number line uses percents; the bottom number line uses fractions. Show students that equivalent numbers appear at the same vertical position; that is, they "line up." For example, the numbers 0.5, 50%, and $\frac{1}{2}$ appear at the same vertical position when the number lines are aligned.
- Ask students how they might use the illustration to compare 0.25 and 50%. On a number line, the number on the left is always less than the number on the right, so 0.25 is less than 50% or 0.25 < 50%.
- Tell students that in today's lesson, they will compare and order numbers, which are given in different forms. Plotting all the numbers on a single number line is one way to visualize the relative size of the numbers.

Subtopic 1 Ordering Fractions, Decimals, and Integers

Expand Their Horizons

In this subtopic, students order fractions, decimals, and integers. Several methods are demonstrated including using models, plotting the numbers on a number line, and using place value to compare.

While drawing or making models of decimal numbers provides an easy visual cue as to how numbers should be ordered, it can be tedious at times. Plotting numbers on a number line to show the numbers in relation to each other also provides an easy visual cue but assumes the student has correctly drawn the number line with an appropriate scale (equal intervals) and correctly identified the location of each number. The scale or equal intervals on a number line may vary depending on the numbers being plotted. The numbers in the problem will provide clues for selecting an appropriate scale. For example, if a student were asked to draw and to plot a point on a number line between zero and 65, the student might select a scale of five and label each hash mark with multiples of five. The relative distance between numbers on a number line can only be compared if an appropriate scale is selected and the numbers correctly plotted. This can be difficult depending on the numbers: for example, comparing $\frac{2}{9}$, 0.21, 0.33, and $\frac{1}{3}$ may be difficult for some students.

The final method for comparing and ordering numbers is to convert each of the numbers to a common form and then to compare. Since comparing decimals is easier in most cases than comparing fractions, students are taught to first convert each number to a decimal number and then to compare.

This lesson requires a level of fluency with the decimal and percent equivalents of benchmark fractions. It may be helpful to review these before viewing the lesson and/or to post a list of these equivalents in the classroom for easy reference.

Take a moment to be sure students understand the number sentence 0.6 > 0.15 > 0.1. This number sentence is called a *compound inequality*, because it states an unequal relationship between more than two quantities. The statement is read "six tenths is greater than fifteen hundredths, and fifteen hundredths is greater than one tenth" or "six tenths is greater than fifteen hundredths is greater than one tenth." Point out that the single statement describes *three* different inequalities at once: 0.6 > 0.15; 0.6 > 0.1; and 0.15 > 0.1. Compound inequalities can be used to list numbers in order from greatest to least (as in 0.6 > 0.15 > 0.1) or from least to greatest (as in -0.25 < -0.2 < 0.25 < 0.29).

1 This problem requires students to compare numbers in decimal, fraction, and integer forms. Point out that three of the numbers are positive, while one is negative. The negative number will be the least. To compare the other three numbers, first write the fraction $\frac{1}{2}$ as the equivalent decimal 0.5. Then, line up all the numbers vertically, making like place values align. Begin comparing with the ones place. Since two is the greatest digit in the ones place, two is the greatest number. Each of the other two numbers has a zero in the ones place and a five in the tenths place. The digits in the hundredths places must be used to compare. $2 > 0.55 > \frac{1}{2} > -1$

2 To show each number on a number line, students should convert (even if mentally) the fraction $\frac{1}{2}$ to its decimal equivalent 0.5. Be sure to check that 0.55 is graphed to the right of 0.5, reminding students that 0.5 = 0.50 and 0.50 < 0.55.
$2 > 0.55 > \frac{1}{2} > -1$

Common Error Alert:

For Lesson Notes Problem 2, students might list the numbers from *least to greatest* rather than *greatest to least*. Remind them to read the direction line carefully before presenting their final answers.

3 Remind students that although each number is converted to decimal form in order to compare, they should return each number to its original form before presenting their final answers. In order from least to greatest, the numbers are $-\frac{1}{5}$, $\frac{4}{5}$, 0.96, and 1.

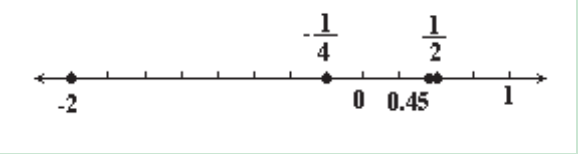
4 When drawing and marking a number line to use for this problem, marking the number line by fifths may be a good choice because each of the numbers $\frac{4}{5}$, $-\frac{1}{5}$, and 1 can be decisively graphed. To graph 0.96, however, students will have to use estimation as marking the number line by hundredths is simply not practical. Remind them that 0.96 lies between 0.8 (decimal equivalent of $\frac{4}{5}$) and 1 and is much closer to 1. Check students' work to be sure they have made a good approximation of 0.96 on the number line. In order from least to greatest, the numbers are $-\frac{1}{5} < \frac{4}{5} < 0.96 < 1$.

Additional Examples

1. Order from least to greatest using a number line.

$\frac{1}{2}$, -2, 0.45, $-\frac{1}{4}$

Graph each number on a number line. Read the numbers as they appear from left to right.



$-2 < -\frac{1}{4} < 0.45 < \frac{1}{2}$

2. Order from greatest to least using place value.

$\frac{1}{2}$, –2, 0.45, $-\frac{1}{4}$

Write each number in decimal form. Compare like place values, beginning with ones.

$\frac{1}{2} = 0.50$

$-2 = -2.00$

$0.45 = 0.45$

$-\frac{1}{4} = -0.25$

$\frac{1}{2} > 0.45 > -\frac{1}{4} > -2$

Subtopic 2 Comparing Fractions, Decimals, and Percents

Expand Their Horizons

In this subtopic, students compare numbers in fraction, decimal, and percent form. Review with the class the procedure for converting between decimals and percents. Remind them that to change a decimal to a percent, the decimal number is multiplied by 100, which moves the decimal point two places to the right. To change a percent to a decimal, divide by 100 or move the decimal point two places to the left.

5 Students may immediately see that $\frac{3}{10}$ is greater than -0.35 because a positive number is always greater than a negative number. Encourage students to graph the numbers on a number line for practice. $\frac{3}{10} > -0.35$

6 The decimal forms of 30% and $\frac{1}{4}$ are 0.30 and 0.25, respectively. Line the numbers up so that like place values can be compared. 0.30 > 0.25 or 0.30 > $\frac{1}{4}$

7 Remind students that 5% means *five hundredths* or 5% = 0.05. To compare 0.4 and 0.05, start by comparing tenths. Four tenths is greater than zero tenths or 0.4 > 0.05.

Additional Examples

1. Compare 0.3 and 10%. Write <, >, or =.

Compare the decimal forms of the numbers.

0.**3**
0.**1**
↓
3 tenths > 1 tenth
0.3 > 0.1
0.3 > 10%

2. Compare 0.65 and $\frac{3}{4}$. Write <, >, or =.

Compare the decimal forms of the numbers.

0.**6**5
0.**7**5
↓
6 tenths < 7 tenths
0.65 < 0.75
0.65 < $\frac{3}{4}$

Look Beyond

Converting among the various forms of numbers and comparing the results are skills which students will use often in future math courses. For example, when the slopes of two lines are compared, it can be determined which line is steeper. If the slope of one line is given as 0.8 and the slope of another line is given as $\frac{3}{4}$, the inequality $0.8 > \frac{3}{4}$ can be used to determine that the first line is steeper.

Connections

Many teachers report test and quiz grades to students in percent form. Students can compare their scores to a "grading scale" that lists the range of percents for each letter grade. One way to encourage students to practice their converting and comparing skills is to give their scores in fraction form. For example, on a quiz with 10 questions of equal weight, a student receiving a $\frac{9}{10}$ must convert the score to 90% and then compare that score with his school's grading scale to determine his letter grade.