Numbers and Operations

Module 5

Decimal Operations, Exponents, and Powers

Lesson 1
Rounding and Comparing Decimals

Teacher Notes 5.1

Objectives

- Round and compare decimals to a given place value (whole number, tenths, hundredths, and thousandths).

Prerequisites

- Reading and writing decimals
- Representing decimals on a number line
- Comparing whole numbers

Vocabulary

- Integers (3.1)
- Decimals (4.2)
- Place value (2.1)
- Mixed number (4.2)
- Improper fractions (4.1)
- Rounding place
- Rounding digit
- Inequality
- Absolute value (3.1)

Get Started

- Prepare for each selected group one set of zip-top bags containing the following amounts in play money: \$0.03 (three pennies), \$0.50 (five dimes), \$2.40 (two dollars, four dimes), and \$2.75 (two dollars, seven dimes, five pennies). Do not label the bags with the amounts.
- Remind students that our system of money is based on a decimal system and is one way to model decimal numbers. For example, the amount \$23.47 represents the sum of two tens, three ones, four tenths, and seven hundredths.
- Hand out the bags containing \$0.03 and \$0.50 to each group. Ask students to open each bag and count the amount of money in each. Then, have them write a statement using <, >, or = to compare the amounts. \$0.03 < \$0.50

- Ask students to focus on the two amounts, \$0.03 and \$0.50. Ask them to compare the amounts using the language of place value rather than the language of money. Three hundredths is less than 50 hundredths. Write 0.03 < 0.50 on the board, point out that it could also be written as 0.03 < 0.5, and read "three hundredths is less than five tenths." Repeat the activity with the bags containing \$2.40 and \$2.75. 2.40 < 2.75, two and 40 hundredths is less than two and 75 hundredths; 2.4 < 2.75, two and four tenths is less than two and 75 hundredths.
- Ask students to suppose that the only forms of currency available are one-dollar bills and dimes (tenths of a dollar). Ask the students to consider whether \$3.72 is closer to three dollars and seven dimes or three dollars and eight dimes.
- Tell students that they will learn how to round and to compare decimals in this lesson. Remind them that when the decimals have at most two decimal places, thinking of the numbers as money amounts can help them round and make comparisons.

Subtopic 1 — Rounding Decimals to a Given Place Value

Expand Their Horizons

This subtopic teaches students how to round and to compare decimals. The methods are similar to those taught in previous lessons on rounding and comparing whole numbers.

Students who are having trouble finding and identifying the rounding place and the digit to its right may benefit from developing a visual cue. Suggest that they underline the rounding digit and circle the place to its right.

The rounding rules state that digits to the right of the rounding place become zeros, and that those zeros can be dropped if they are to the right of the decimal point.
There are many reasons to round a decimal number. As with whole numbers, rounding decimal numbers can make them easier to interpret. For example, being told that the distance between two landmarks is 53.84303 miles is generally no more useful than being told that the distance is 54 miles. Often decimal numbers are rounded when dealing with money, such as when a unit price of \$3.2391 is rounded to the nearest cent. In addition, rounded numbers are used to make estimates.

1 To round 4.81 to the nearest tenth, locate the digit in the tenths place (eight). Since the digit in the hundredths place is one, the digit in the rounding place does not change. All digits to the right become zero. Therefore, 4.81 rounded to the nearest tenth is 4.8.

2 The digit in the rounding place is two. The digit to its right is eight. The rounding digit is increased by one, and all digits to its right become zero. So, 0.428 rounded to the nearest hundredth is 0.430, or 0.43.

3 Remind students that rounding to the "nearest whole number" is rounding to the ones place. So, the ones place contains the rounding digit. The number 38.573 rounds to 39.

Common Error Alert:

Caution students to be careful not to drop zeros in whole numbers and zeros used as place holders in the decimal part of the number. For example, when 185 is rounded to the nearest hundred, the zeros in 200 obviously can not be dropped. However, when 0.185 is rounded to the nearest tenth, the zeros in 0.200 can be dropped, since 0.200 has the same value as 0.2. When 3.0486 is rounded to the nearest hundredth, the result is 3.05. The zero in the tenths place cannot be dropped.

Additional Examples

1. **Round 8.382 to the nearest hundredth.**

   The digit in the rounding place is eight. The digit to its right is two.

   $8.382 \rightarrow 8.380 \rightarrow 8.38$

2. **Round 0.74 to the nearest whole number.**

   The digit in the rounding place is zero. The digit to its right is seven.

   $0.74 \rightarrow 1.00 \rightarrow 1$

Subtopic 2 — Comparing Positive Decimals

Expand Their Horizons

In Subtopics 2 and 3, students compare decimal numbers. This lesson breaks the examples into two subtopics: comparing two positive decimals and comparing two negative decimals. Be sure to point out that a positive decimal can be compared to a negative decimal, and that a positive decimal is always greater than a negative decimal.

Emphasize the use of the word *inequality* during this lesson. An inequality is a number sentence that states that two quantities are *not* equal. Contrast inequalities with equations, which are number sentences stating that two quantities *are* equal. The number sentence 2 + 2 = 4 is an equation; the number sentence 2 + 3 > 4 is an inequality. Encourage students to use the term *inequality*. They will study these types of number sentences in depth in the future.

When comparing decimal numbers, it can be useful to rewrite the numbers so that they have the same number of decimal places. This can be done by adding zeros to the end of the decimal part of one of the numbers. Once this is done, reading the numbers aloud can provide an auditory cue for comparing. For example, when comparing 3.9 and 3.27, writing 3.9 as 3.90 allows the numbers to be read as “three and 90 hundredths” and “three and 27 hundredths.”

For each exercise and example in which the two numbers are not equal, remind students that there are two correct ways to write the answer. For example, 0.054 < 0.09 can also be written 0.09 > 0.054. Both represent the same inequality but are read differently. The examples on the DVD show the inequality written such that the first number presented in the problem is written first in the inequality. Assure students that either way is correct.

Visual learners and students having trouble comparing decimal fractions might benefit from the use of models. When comparing numbers through hundredths, they can model each number using hundredths-squares and then can decide which model shows the greater amount.

4 The numbers are identical in each place value: 4.25 = 4.25. Point out that this comparison resulted in an equation, rather than an inequality.

5 The numbers are identical in the ones and tenths places. In the hundredths place, 0.05 < 0.09, so 0.054 < 0.09.

Common Error Alert:

Students may count the number of nonzero digits or the number of decimal places to determine which number is greater. They may think 0.054 is greater than 0.09 because there are three decimal places in 0.054 and only two in 0.09 (or because there are two nonzero digits in 0.054 and only one in 0.09, or because 54 > 9). Remind them to be careful when comparing numbers and to follow the rules outlined on the DVD.

Additional Examples

1. Use <, >, or = to compare the decimals.

3.28 and 3.7

Starting with the ones place, compare the digits.

3.28 < 3.7

2. Use <, >, or = to compare the decimals.

0.012 and 0.004

Starting with the ones place, compare the digits.

0.012 > 0.004

Expand Their Horizons

Subtopic 3 teaches students how to compare two negative decimal numbers. Before comparing negative decimal numbers, be sure students have mastered the skill of comparing negative integers. Just as with positive integers, when two negative numbers are shown on a number line, the greater number always lies to the right. Despite the rule, students are often distracted by the absolute value of a number. They may think -100 is greater than -2 because 100 > 2. To help them avoid this mistake, prepare a list of inequalities that compare two negative integers. (e.g. -3 ☐ -7). Ask them to visualize the integers on the number line as they complete each inequality using <, >, or =.

To compare negative decimals, students are taught to compare absolute values. The number with the greater absolute value lies farther to the left of zero than the number with the lesser absolute value, so it is the lesser number.

Some students may wonder whether they can use place value to compare negative decimals, as they did when comparing positive decimals. Demonstrate to those students that they can compare two negative decimals by comparing like place values until the digits are different. The lesser digit is in the greater number. Contrast this algorithm with the one used for comparing positive decimals, where the lesser digit is in the lesser number.

6 Find the absolute value of each number. The number with the greater absolute value is the lesser number, so -76.3 < -67.2. The inequality can also be written -67.2 > -76.3.

7 Rewrite -8.1 as -8.100. Since |-8.001| < |-8.100|, -8.1 < -8.001.

Common Error Alert:

Students may ignore the negative signs and compare only absolute values. They may write 8.1 > 8.001. Remind them to check the signs carefully and take them into account when comparing numbers.

Additional Examples

1. **Use <, >, or = to compare the decimals.**

-7.5 and -7.054

Compare absolute values. The number with the greater absolute value is the lesser number.

$|-7.5| = 7.5$
$|-7.054| = 7.054$

$|7.5| > |7.054|$
$-7.5 < -7.054$

2. **Use <, >, or = to compare the decimals.**

-5.26 and -5.29

Compare absolute values. The number with the greater absolute value is the lesser number.

$|-5.26| = 5.26$
$|-5.29| = 5.29$

$|5.26| < |5.29|$
$-5.26 > -5.29$

Look Beyond

Comparing numbers is a fundamental skill of mathematics. When two numbers are compared, there are three possibilities: the numbers are equal, the first number is greater than the second, or the first number is less than the second. This law, called the Law of Trichotomy, is fundamental to mathematics because it guarantees that two quantities can not be both equal and unequal at the same time.

Connections

Many everyday measurements require comparing and rounding decimal numbers. For example, batting averages are represented by decimals from zero to one and are generally rounded to the nearest thousandth. A player with 745 hits out of 2000 times at bat has a 745 ÷ 2000, or a 0.3725 batting average. This average is rounded to the nearest thousandth to get a batting average of 0.373. Batting averages are compared to judge one player against another or to study a player's progress over previous years' statistics.