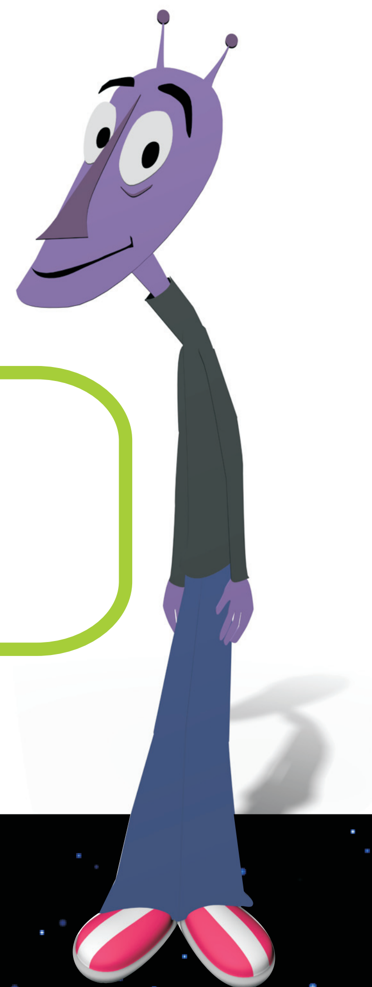
Numbers and Operations

Module 4

Fractions, Decimals, Percents, and Factors



Lesson 5
Simplifying and Converting Fractions

Teacher Notes 4.5

Objectives

- Use factors of numbers to simplify fractions to the lowest terms.
- Convert between mixed numbers and improper fractions.

Prerequisites

Reading, writing, and modeling fractions

Multiplying and dividing whole numbers

Vocabulary

Rational number
Equivalent fractions (4.1)
Fraction (4.1)
Simplify
Convert
Numerator (4.1)
Denominator (4.1)
Mixed number (4.2)
Integer (3.1)
Proper fraction (4.1)
Improper fraction (4.1)
Lowest terms
Greatest Common Factor (4.3)
Relatively prime
Simplest form

Get Started

- Divide the class into groups. Give each group a stack of semi-circles cut from paper.
- Ask students to identify the fraction of a whole circle modeled by the semi-circle. $\frac{1}{2}$ Next, ask the group to count out seven semi-circles. Point out that they have “seven halves” and ask them to put the halves together to form as many wholes as possible. **Students can make three wholes with one half left over.** Ask a group to share their model and ask the class what mixed number can be used to name the quantity modeled. **$3\frac{1}{2}$** Point out that seven halves is the same as three wholes and one half. Write the equation $\frac{7}{2} = 3\frac{1}{2}$ on the board.
- Next, ask students to use the semi-circles to model the number four. Ask them how many halves are needed. **eight halves** Write the equation $4 = \frac{8}{2}$ on the board.

- Finally, ask them to model the mixed number $2\frac{1}{2}$ and identify how many halves are needed. **five halves** Write the equation $2\frac{1}{2}=\frac{5}{2}$ on the board.
- Point out the three equations on the board and tell students that each one shows an improper fraction which is equal to a mixed number or a whole number. Tell them that this lesson will teach them how to convert among these forms.

Subtopics 1 & 2

Rational Numbers and Equivalent Fractions and Simplifying Fractions to Lowest Terms

Expand Their Horizons

The first subtopic introduces the term rational number. A rational number is a number that can be written as the ratio of two integers *a* and *b*, where *b* cannot be zero. Tell students that almost every number they have encountered so far in mathematics is a rational number. Remind students that a number is a rational number if it can be written in the form $\frac{a}{b}$ $(b \neq 0)$, not if it is written in that form. For example, the number 4.3 is not written in the required form, but it could be: $4.3 = 4\frac{3}{10} = \frac{43}{10}$.

So far, the only irrational numbers they have most likely encountered have been square roots of certain numbers. For example, $\sqrt{2}$ is an irrational number because there is no way to write the number in the form $\frac{a}{b}$ $(b \neq 0)$. Infinite non-repeating decimals (such as 0.313113111311113…) are irrational numbers.

When writing equivalent fractions, the terms *simplest form* and *lowest terms* are used interchangeably. Have students consider these terms. Ask them why they think the expressions simplest form and lowest terms are used.

1 The GCF of 14 and 28 is 14. Write the numerator as the product of the GCF and another factor. Do the same for the denominator. Then, divide out the GCF. $\frac{14}{28} = \frac{\overset{1}{\cancel{14}}\cdot 1}{\underset{1}{\cancel{14}}\cdot 2} = \frac{1}{2}$

Common Error Alert:

Students may find a common factor of 14 and 28 that is not the Greatest Common Factor. For example, they may write $\frac{14}{28} = \frac{\overset{1}{\cancel{2}}\cdot 7}{\underset{1}{\cancel{2}}\cdot 14} = \frac{7}{14}$. Remind them to be sure the numerator and denominator have no common factors.

2 The GCF of 24 and 36 is 12. Write the numerator and denominator as products using the GCF as a factor and then divide out the GCF. $\frac{24}{36}=\frac{\overset{1}{\cancel{12}}\cdot 2}{\underset{1}{\cancel{12}}\cdot 3}=\frac{2}{3}$

3 The GCF of 12 and 40 is four. $\frac{12}{40}=\frac{\overset{1}{\cancel{4}}\cdot 3}{\underset{1}{\cancel{4}}\cdot 10}=\frac{3}{10}$

Additional Examples

1. Write $\frac{15}{24}$ in simplest form.

Write the numerator and denominator using products of the GCF.

$$\frac{15}{24}=\frac{\overset{1}{\cancel{3}}\times 5}{\underset{1}{\cancel{3}}\times 8}=\frac{5}{8}$$

$$\frac{5}{8}$$

2. Write $\frac{16}{48}$ in simplest form.

Write the numerator and denominator using products of the GCF.

$$\frac{16}{48}=\frac{\overset{1}{\cancel{16}}\cdot 1}{\underset{1}{\cancel{16}}\cdot 3}=\frac{1}{3}$$

$$\frac{1}{3}$$

Subtopic 3 Converting an Improper Fraction to a Mixed Number

Expand Their Horizons

In this subtopic, students are shown how to convert an improper fraction to the equivalent mixed number. Every improper fraction has a mixed number or whole number equivalent, and vice versa.

One of the examples on the DVD asks students to write $\frac{26}{4}$ as a mixed number. Some students might notice that $\frac{26}{4}$ is an improper fraction that is not in simplest form. One way to approach the problem is to first write the fraction in simplest form and then convert to a mixed number. $\frac{26}{4}=\frac{\overset{1}{\cancel{2}}\cdot 13}{\underset{1}{\cancel{2}}\cdot 2}=\frac{13}{2}=6\text{ R}1=6\frac{1}{2}$ Since the mixed number equivalent is unique, either method is valid.

4 Point out that $1 = \frac{3}{3}$. $\frac{10}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = 1 + 1 + 1 + \frac{1}{3} = 3\frac{1}{3}$

5 Each whole is five fifths. Nineteen fifths can make three wholes with four fifths left over. $\frac{19}{5} = 3\frac{4}{5}$

6 Divide 65 by nine. Use the quotient as the integer part of the mixed number, the remainder as the numerator of the fraction part, and the divisor nine as the denominator of the fraction part. $\frac{65}{9} = 65 \div 9 = 7\text{ R}2 = 7\frac{2}{9}$

Additional Examples

1. Write $\frac{29}{8}$ as a mixed number.

Determine the number of wholes that can be made with 29 eighths. Make a fraction using the remaining eighths.

$$\frac{29}{8} = 29 \div 8 = 3\text{ R}5 = 3\frac{5}{8}$$

$$3\frac{5}{8}$$

2. Write $\frac{68}{10}$ as a mixed number.

Divide the numerator by the denominator. Use the quotient as the whole number part, the remainder as the numerator, and the divisor 10 as the denominator.

$$\frac{68}{10} = 68 \div 10 = 6\text{ R}8 = 6\frac{8}{10}$$

$$6\frac{8}{10} \text{ or } 6\frac{4}{5}$$

Subtopic 4 Converting Mixed Numbers to Improper Fractions

Expand Their Horizons

This subtopic shows how to do the operation studied in the previous subtopic in reverse. Knowing how to convert between the two forms enables students to check their work. They can convert an improper fraction to a mixed number and then convert the mixed number back to an improper fraction to check the answer.

7 Students can think of $2\frac{1}{3}$ as two wholes plus $\frac{1}{3}$. Since each whole is $\frac{3}{3}$, $2\frac{1}{3} = 1 + 1 + \frac{1}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{7}{3}$. Using another method, $2\frac{1}{3} = \frac{3\times2+1}{3} = \frac{7}{3}$.

8 Multiply the denominator by the whole number part and then add the numerator. The result is the numerator of the improper fraction. Use the denominator from the mixed number as the denominator of the improper fraction. $4\frac{3}{5} = \frac{5\times4+3}{5} = \frac{23}{5}$

Additional Examples

1. **Write $3\frac{7}{8}$ as an improper fraction.**

To find the numerator, find 8 × 3 + 7. The denominator is eight.

$$3\frac{7}{8}=\frac{8\times3+7}{8}=\frac{31}{8}$$

$$\frac{31}{8}$$

2. **Write $5\frac{2}{7}$ as an improper fraction.**

Each whole is $\frac{7}{7}$, so five wholes is $\frac{35}{7}$. Add $\frac{35}{7}+\frac{2}{7}$ to find the improper fraction equivalent.

$$5\frac{2}{7}=\frac{7\times5+2}{7}=\frac{37}{7}$$

$$\frac{37}{7}$$

Look Beyond

Knowing how to convert among the various forms of a number (e.g. mixed number, improper fraction, or decimal) will become important as students begin to apply number operations. For example, when multiplying two mixed numbers, each must be converted to an improper fraction before the multiplication can be carried out. Given two improper fractions to compare, the task can be much simpler if the improper fractions are converted to mixed numbers. Students will compare and perform operations on rational numbers in future lessons of this course.

Connections

Expressing fractions in simplest form helps students gain an intuitive understanding of the portion being described. For example, a principal knows that $\frac{492}{656}$ of the students in his school scored above average on a standardized test. The portion of students in the school who scored above average on the standardized test is more clearly expressed by stating the fraction in its simplest form, $\frac{3}{4}$.