

Numbers and Operations

★ Module 4 ★

Fractions, Decimals, Percents, and Factors

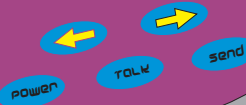
Lesson 4 Prime Factorization, GCF, and LCM



Teacher Notes 4.4

Objectives

- ♦ Use a variety of methods, including prime factorization, to determine the Least Common Multiple (LCM).
- ♦ Apply factorization, GCF, and LCM to solve problems.



Prerequisites

Multiplying and dividing whole numbers
Finding the prime factorization of a number
Finding the GCF of two or three numbers

Vocabulary

Least Common Multiple (2.3)
Product (2.3)
Counting number
Greatest Common Factor (4.3)
Prime factorization (4.3)
Factor (1.2)

Get Started

- Review the term multiple. A multiple is the product of a number and a counting number (1, 2, 3, etc.).
- Divide the class into groups of four with two pairs in each group. Prepare copies of a piece of paper divided with lines into four rows and nine columns with the counting numbers from 1 to 9 written in each row, one number per column. Give each group a copy and ask them to cut the paper into pieces along the lines. They should have 36 pieces with four copies of each counting number.
- Direct the groups to place all 36 pieces of paper in a hat or box. Each student selects one piece of paper and names the number shown on the card and the multiples of the number that are found by multiplying the number by 1, 2, 3, 4, and 5. If a player selects “two,” she says, “Two, four, six, eight, 10.” If her partner selects “three,” he says, “three, six, nine, 12, 15.” The players identify whether they have found any common multiples. If there are none, the score for that play is zero. After each play, the pieces are discarded and play repeats.

- If there are common multiples, the players identify the least of the common multiples and record it as their score. The first team to reach 30 points wins. (Point out that when partners draw the same number, the multiples are the same, and the least of the common multiples is the number itself).
- Tell students that they have been finding the Least Common Multiple of two numbers.

Subtopic 1

Least Common Multiple

Expand Their Horizons

This subtopic introduces students to the Least Common Multiple, or LCM. The Least Common Multiple of two or more numbers is the smallest of their common multiples. To find the multiples of a number, multiply the number by each counting number. Remind students that the counting numbers (also known as the natural numbers) are the set of numbers they would naturally name if they were asked to count: 1, 2, 3, 4...

Point out that the counting numbers do not include zero. For any number n , the number $n \times 1$, or n itself, is the first multiple of n .

In this subtopic, students find the LCM by listing the multiples of each number. Some students may realize that using this method to find the LCM of larger numbers, such as 312 and 254, would be a lengthy process. Assure them that a more efficient method will be taught in the next subtopic.

One “shortcut” students might suggest is to list only the multiples of the greater number and then check to see whether each of those multiples is a multiple of the smaller number. For instance, in Lesson Notes Problem 1, they might list the first several multiples of 15: 15, 30, 45, 60, 75 and then test each one to determine whether it is a multiple of nine. This is a valid method and demonstrates good number sense.

1

List the multiples of each number. The Least Common Multiple is 45.

2

List the multiples of each number. The Least Common Multiple is 72.

Additional Examples

1. Find the LCM of 10 and 12.

List the multiples of 10 and the multiples of 12. Identify the Least Common Multiple.

10: 10, 20, 30, 40, 50, **60**, 70, 80

12: 12, 24, 36, 48, **60**, 72, 84,

LCM = 60

2. Find the LCM of 8, 12, and 15.

List the multiples of each number. Identify the least number common to all three lists.

8: 8, 16, 24...88, 96, 104, 112, **120**

12: 12, 24, 36...84, 96, 108, **120**

15: 15, 30, 45, 60, 75, 90, 105, **120**

LCM = 120

Expand Their Horizons

In this subtopic, students are shown how to use the prime factorizations of two or more numbers to determine their Least Common Multiple. The Least Common Multiple must contain all factors of each number. When prime factorizations are written in exponential notation, the LCM is the product of the greatest power of each base appearing in the prime factorizations. Therefore, if 12 is written as $2^2 \times 3$ and 16 is written as 2^4 , the bases used are two and three. The product of the greatest powers of the bases is $2^4 \times 3$ or 16×3 or 48.

Common Error Alert:

Students may have difficulty keeping track of which prime factors are included in the LCM product. The DVD shows the factors written so that factors are lined up in columns, with the common factors lined up. Students struggling with using prime factorization to find the LCM might benefit from using graph paper to mimic the method shown on the DVD. Remind them that each column should contain only one number. For example, a column should contain only twos, only threes, etc.

3

The prime factorization of eight is $2 \times 2 \times 2$. The prime factorization of 27 is $3 \times 3 \times 3$. Since there are no common factors, the LCM is $2 \times 2 \times 2 \times 3 \times 3 \times 3$, or 216. Explain to students that sometimes the LCM of a set of numbers is the product of the two numbers. Identifying the conditions under which this occurs is an exercise in the Challenge Problems.

4

The prime factorization of 28 is $2^2 \times 7$. The prime factorization of 42 is $2 \times 3 \times 7$. The two numbers share the common factors two and seven. The non-common factors include one 2 and one 3. So, the LCM is $2 \times 2 \times 3 \times 7$, or 84.

5

The prime factorizations of 12, 16, and 24 are, respectively, $2 \times 2 \times 3$, $2 \times 2 \times 2 \times 2$, and $2 \times 2 \times 2 \times 3$. The greatest number of times two is used as a factor is four times; the greatest number of times three is used is one time. The resulting LCM is $2 \times 2 \times 2 \times 2 \times 3$, or 48.

In Lesson Notes Problem 5, students may notice that 24 is a multiple of 12. Since any multiple of 24 is also a multiple of 12, it is possible to disregard the number 12 when calculating the LCM; the LCM of 16 and 24 will also be the LCM of 12, 16, and 24.

Additional Examples

1. Find the LCM of 32 and 42.

Find the product of the greatest number of times each prime factor is used.

$$\begin{array}{l} 32: 2 \times 2 \times 2 \times 2 \times 2 \\ 42: 2 \qquad \qquad \times 3 \times 7 \end{array}$$

The factor 2 is used five times. The factors 3 and 7 are used once, respectively.

The LCM is $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 7$, or 672.

2. Find the LCM of 15, 18, and 24.

Find the product of the greatest number of times each prime factor is used.

$$\begin{array}{l} 15: \qquad \qquad \qquad 3 \times 5 \\ 18: 2 \qquad \qquad \times 3 \times 3 \\ 24: 2 \times 2 \times 2 \times 3 \end{array}$$

The factor 2 is used three times; 3 is used twice, and 5 is used once.

The LCM is $2 \times 2 \times 2 \times 3 \times 3 \times 5$, or 360.

Subtopics 3 & 4

Applications Using the LCM and GCF of More Than Two Numbers

Expand Their Horizons

These subtopics show students problems that can be solved using the LCM and the GCF.

Common Error Alert:

Students may be confused about when to use the LCM and when to use the GCF when solving word problems. As the students watch the DVD, discuss the similarities between the problems. Guide them to identify that the Least Common Multiple is generally used when two or more items (e.g. Trains Alpha, Beta, and Gamma; beads costing 12, 36, and 52 moon dollars each) repeat until they are equal in some way (e.g. time; price). The Greatest Common Factor is generally used when two or more groups of items (e.g. Neptunians, Martians, and Venusians; pencils, pens, and markers) are divided into equal groups (e.g. rows, supply boxes) containing as many items as possible.

6

The LCM of 12, 36, and 52 shows the least amount (in moon dollars) that can be spent on beads while spending an equal amount on each type of bead. The prime factorizations, respectively, are $2 \times 2 \times 3$; $2 \times 2 \times 3 \times 3$; and $2 \times 2 \times 13$. Therefore, the LCM is $2 \times 2 \times 3 \times 3 \times 13$, or 468. Since 468 moon dollars are spent on each color, the total cost of the necklace is 468×3 , for a total of 1,404 moon dollars.



The GCF of 20, 30, and 45 shows the greatest number of supply boxes that can be made using 20 pencils, 30 pens, and 45 markers if an equal number of each item is used in each box. The prime factorizations, respectively, are $2 \times 2 \times 5$; $2 \times 3 \times 5$; and $3 \times 3 \times 5$. The Greatest Common Factor is five. Since five supply boxes can be made, divide the available number of each item by five to find the number of each item in each box. Each supply box contains four pencils, six pens, and nine markers. Each supply box contains $4 + 6 + 9$, or 19 items.



The GCF of 32, 36, and 40 shows the greatest number of identical flower arrangements that can be made using 32 carnations, 36 roses, and 40 daisies. The prime factorizations, respectively, are $2 \times 2 \times 2 \times 2 \times 2$; $2 \times 2 \times 3 \times 3$; and $2 \times 2 \times 2 \times 5$. The Greatest Common Factor is four. Since four identical flower arrangements can be made, divide the available number of each flower by four to find the number of each flower in each arrangement. Each arrangement contains eight carnations, nine roses, and 10 daisies. To find the total cost of each arrangement, multiply the number of each flower times the cost per flower and add those products. $(8 \times 1) + (9 \times 4) + (10 \times 2) = 64$. The total cost of each arrangement is \$64.

Additional Examples

1. To make a trail mix, George needs equal amounts (by weight) of pretzels, spacenuts, and chocolate candies. A 4. 12-ounce bag of pretzels costs \$2.29, a 5. 24-ounce bag of spacenuts costs 6. \$3.28, and a 20-ounce bag of chocolate 7. candies costs \$4.50. What is the least 8. amount George can spend to get an 9. equal weight of each ingredient?

Find the LCM of 12, 24, and 20. The LCM shows the weight of each ingredient George must buy. Divide to find the number of packages and then multiply to find the price of each ingredient. Add to find the total cost.

LCM = 120

Ten packages of pretzels cost \$22.90.

Five packages of spacenuts cost \$16.40.

Six packages of chocolate candies cost \$27.00.

$\$22.90 + \$16.40 + \$27.00 = \66.30

George must spend at least \$66.30 to get an equal weight of each ingredient.

2. Stef has 56 blueberries, 42 strawberries, and 35 raspberries. She wants to evenly distribute the three kinds of berries onto as many plates as possible. Each plate must have each kind of berry with no berries left over. How many berries will be on each plate?

Find the GCF of 56, 42, and 35. The GCF shows the number of plates. Divide to find the number of each kind of berry on each plate and then add to find the total number of berries on each plate.

GCF = 7

$56 \div 7 = 8$ blueberries

$42 \div 7 = 6$ strawberries

$35 \div 7 = 5$ raspberries

$8 + 6 + 5 = 19$

There will be 19 berries on each plate.

Look Beyond

When adding two or more fractions, each fraction must have the same denominator. One way to give the fractions the same denominator is to use the LCM of the denominators to create equivalent fractions. Since the LCM of six and nine is 18, the Least Common Denominator, or LCD, of $\frac{1}{6}$ and $\frac{2}{9}$ is 18, and the problem $\frac{1}{6} + \frac{2}{9}$ can be rewritten as $\frac{3}{18} + \frac{4}{18}$. The sum is $\frac{7}{18}$. In algebra, the LCM of the expressions $3xy$ and $9x^2$ is $9x^2y$, and the LCM can be used to transform the sum $\frac{1}{3xy} + \frac{2}{9x^2}$ into $\frac{3x}{9x^2y} + \frac{2y}{9x^2y}$, which can be simplified to $\frac{3x+2y}{9x^2y}$.

Calculator

Students may be interested to see that some calculators can be used to find the GCF and LCM of a set of numbers. Many graphing calculators have GCF and LCM functions built in. If a graphing calculator is available, demonstrate the use of the GCF and LCM functions for students. [On the TI-83, select the NUM tab in the MATH menu. Then, scroll down to option eight (LCM) or nine (GCF) and select. When prompted, enter two numbers separated by a comma. Press the ENTER key to find the desired value.]

There are also several applets for finding GCF and LCM available online. Enter "LCM applet" or "GCF applet" into an internet search engine and let students experiment with finding the LCMs and GCFs needed to solve the problems from this lesson.