Numbers and Operations

Module 4

Fractions, Decimals, Percents, and Factors

Lesson 3
Factors and Prime Factorization

Teacher Notes 4.3

Objectives

- Find the factors of a number.
- Determine if a number is prime or composite.
- Find the prime factorization of a composite number.
- Use factors of a number to find common factors of two integers, including finding the Greatest Common Ffactor (GCF) of two or more integers.
- Use prime factorization to determine the Greatest Common Factor (GCF).

Prerequisites

Understanding and using Venn diagrams
Multiplying and dividing whole numbers
Writing expressions using exponents; expanding exponents
Comparing whole numbers
Factoring using arrays

Vocabulary

Factor (1.2)
Remainder (1.2)
Product (1.1)
Prime number
Composite number
Even number (1.2)
Odd number (2.5)
Factorization
Prime factorization
Array
Factor ladder
Factor pair
Factor tree
Expanded form of prime factorization
Exponential form of prime factorization
Common factor
Greatest Common Factor
Venn diagram (2.5)

Get Started

- Divide the class into groups. Give each group 24 counters.
- Tell the groups to use the counters to form an array with an equal number of rows and an equal number of columns. **Possible arrays are 1-by-24; 2-by-12; 3-by-8; and 4-by-6, and those found by changing the order of the dimensions. (A 3-by-8 array and an 8-by-3 array are considered the same for the purposes of this activity.)**
- Ask the groups to name the multiplication problem modeled by their arrays. **Possible answer: a 2-by-12 array models the product 2 × 12.** Point out that every group did not create the same array with their 24 counters. Write each of the different products modeled on the board, asking students to consider whether any other models might have been possible.

- Next, collect one counter from each group, leaving 23 counters. Again, ask students to make an array. Ask volunteers to name the product modeled by their arrays. **All students should answer either 23 × 1 or 1 × 23.**
- Ask students to identify differences between the numbers 24 and 23. Guide them to see that there were several ways to make an array with 24 but only one way with 23. Tell them that this lesson will teach them how to classify a number as prime or composite according to the number of ways that counters can be arranged in arrays.

Subtopic 1 — Finding the Factors of a Number

Expand Their Horizons

This subtopic teaches students how to find the factors of a number and how to classify a number as either prime or composite.

Review the terms factor and product with students. It may be helpful to write the expression “factor × factor = product” on the board or elsewhere in the classroom for reference.

Before doing the exercises in which students have to list the factors of a number, review the divisibility rules with the class. Fluency with these rules will make finding factors much easier for students. Here is a quick review.

2: The number ends in 2, 4, 6, 8, or 0.
3: The sum of the digits is a multiple of 3.
4: The number formed by the tens and ones places is a multiple of 4.
5: The number ends in 0 or 5.
6: The number is divisible by both 2 and 3.
8: The number formed by the hundreds, tens, and ones places is a multiple of 8.
9: The sum of the digits is a multiple of 9.

1 Twenty-one can not be divided equally into groups of six. Since there is a remainder (three), six is not a factor of 21.

2 Use divisibility rules and basic factor pairs to find the factors. The factors of 100 are 1, 2, 4, 5, 10, 20, 25, 50, and 100.

3 Nine has factors other than one and nine (for example, three). So, nine is composite. Since 56 has factors other than one and 56 (for example, two), 56 is composite. Twenty-nine has no factors other than one and 29, so 29 is prime.

Additional Examples

1. List the factors of 32.

Find factors by listing factor pairs.
1 x 32, 2 x 16, and 4 x 8

The factors are 1, 2, 4, 8, 16, 32.

2. Prime or composite?
16, 37, 81

Prime numbers have no factors other than one and themselves. Composite numbers have more than two factors.

16: composite (There are four factors.)
37: prime
81: composite (There are three factors.)

Subtopic 2 Finding the Prime Factorization of a Number

Expand Their Horizons

In the previous subtopic, students found the factors of a number by first finding all the number's factor pairs. Each of these pairs can also be used to write a factorization of the number. In this subtopic, prime factorization is studied.

At this point in mathematics, students have seen a few different uses of the word factor. Used as a noun, it is one of the numbers used in a multiplication expression (In $4 \times 3 = 12$, four and three are factors.) It may also refer to a number that divides another number with no remainder. (Four is a factor of 32.) In this section, factor is used as a verb. To factor a number is to write it as the product of two or more numbers. The alternate verb form factorize is used to form the term prime factorization, which shows the number written as the product of prime factors.

Students are introduced to two different constructs for finding prime factorization: a factor tree and a "ladder" diagram. Remind students that either construct should produce the same prime factorization since the prime factorization of a number is unique. Students may prefer one method over the other; encourage them to develop a level of comfort with each one. Students may notice that some factor trees are shorter than others. Tell them, that each time they select a pair of factors, the less difference between the factors, the shorter their tree will be. So, when finding the prime factorization of 24, an initial selection of four and six will yield a shorter tree than an initial selection of two and 12.

4 Remind students that their factor trees may look different than the one shown in the DVD. Consider asking volunteers to show their different factor trees on the chalkboard, emphasizing that the end results are the same. The prime factorization of 48 is $2^4 \times 3$.

Common Error Alert:

Students sometimes stop factoring too soon. For example, they may write the prime factorization of 48 as 2 × 3 × 2 × 4. Remind them to double-check their prime factorizations to be sure each factor is prime.

5 The prime factorization of 98 is 2×7^2.

6 The prime factorization of 150 is $2 \times 3 \times 5^2$.

Common Error Alert:

When using the ladder diagram, students may forget to include the last number in the right-hand column in the prime factorization. They may write 2 × 7 instead of 2 × 7 × 7. Help them avoid this error by telling them to draw an L-shaped line that passes through all the numbers in the left-hand column and then turns right to pass through the bottom number of the right-hand column.

Additional Examples

1. Find the prime factorization of 160.

Make a factor tree starting with one pair of factors.

```
        160
       /   \
    10  ×  16
   / \     / \
 2 × 5  × 4  ×  4
 /   /   / \   / \
2 × 5 × 2 × 2 × 2 × 2
```

$2^5 \times 5$

2. Find the prime factorization of 90.

Make a factor ladder.

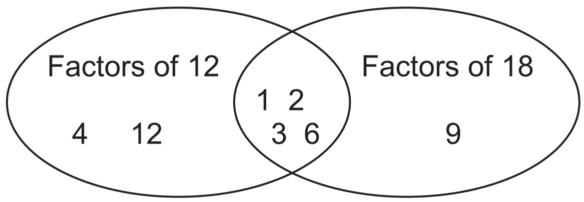
	90
3	30
2	15
3	**5**

$2 \times 3^2 \times 5$

Expand Their Horizons

In this subtopic, students study common factors and the Greatest Common Factor (GCF).

In preparation for the next subtopic, a review of Venn diagrams may be in order. A Venn diagram uses intersecting circles to show sets. The elements common to both sets appear in the intersection. A Venn diagram can be used to find the common factors of 12 and 18 by placing the factors of 12 in one circle and the factors of 18 in the other circle. Point out that one, two, three, four, six, and 12 lie in the left-hand circle; one, two, three, six, and nine lie in the right-hand circle; and one, two, three, and six lie in both circles. Therefore, one, two, three, and six are the common factors of 12 and 18.



7 List the factors of each number and then find the factors common to both lists. The common factors of 24 and 60 are one, two, three, four, six and 12.

8 The common factors of 24 and 60 are one, two, three, four, six, and 12. The greatest common factor is 12.

Additional Examples

1. Find the common factors of 36 and 60.

List the factors of each number. Find the factors common to both lists.

36: 1, 2, 3, 4, 6, 9, 12, 18, 36
60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Common factors: 1, 2, 3, 6, 12

2. Find the GCF of 36 and 60.

Look at the list of common factors. Find the Greatest Common Factor.

Common factors: 1, 2, 3, 6, 12

Greatest Common Factor: 12

Expand Their Horizons

This subtopic shows students one of the uses of prime factorization: finding the GCF of two or more numbers. To find the GCF, find the prime factorizations of each number in expanded form. The GCF is the product of the common prime factors.

When the prime factorizations are written in exponential form, the GCF is the product of the least power of each common base (for example, $72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$). The common bases are two and three. The product of the least power of the common bases is 2×3^2, or 2×9, or 18.

9 The common prime factors are five and five. The GCF is 25.

Common Error Alert:

Students may count the common base, five, only once. Remind them that each instance of a common base counts toward the GCF.

10 In this exercise, students find the GCF of three numbers. Remind them that the common bases have to be common to all three numbers. If a factor is common to only two of the numbers, it is not used in finding the GCF. The common prime factors of 98, 70, and 42 are two and seven. So, the GCF is 2 × 7, or 14.

Additional Examples

1. **Find the GCF of 76 and 114.**

   Find the product of the common prime factors.

   $$\begin{aligned} 76 &= \boxed{2} \times 2 \times \boxed{19} \\ 114 &= \boxed{2} \times 3 \times \boxed{19} \end{aligned}$$

   GCF: 2 × 19 = 38

2. **Find the GCF of 60, 108, and 36.**

   Find the product of the common prime factors.

   $$\begin{aligned} 60 &= \boxed{2} \times \boxed{2} \times \boxed{3} \times 5 \\ 108 &= \boxed{2} \times \boxed{2} \times \boxed{3} \times 3 \times 3 \\ 36 &= \boxed{2} \times \boxed{2} \times \boxed{3} \times 3 \end{aligned}$$

   GCF: 2 × 2 × 3 = 12

Look Beyond

In this lesson, students learned how to factor a number by writing it as the product of two or more numbers. In the next lesson, students will learn how to use the GCF to simplify fractions. In algebra, expressions are often factored to simplify fractions. For example, the fraction $3x–9/6x–21$ can be simplified to $3(x–3)/3(2x–7) = (x-3)/(2x-7)$ because the GCF(three) can be divided out of both the numerator and denominator. Knowing how to find the Greatest Common Factor of two or more numbers is essential to the successful factoring of algebraic expressions.

Connections

The concept of Greatest Common Factor has many applications in everyday life. These applications typically involve situations in which two or more collections of objects must be divided into equal groups that are as large as possible. For example, when a grocer has 64 apples and 72 oranges and wants to bundle the fruit into identical displays, he finds the GCF of 64 and 72. The GCF, eight, tells him that he can make eight identical displays, each containing 64 ÷ 8, or eight apples, and 72 ÷ 8, or nine oranges.