Numbers and Operations

Module 4

Fractions, Decimals, Percents, and Factors

Lesson 2
Concepts of Decimal Place Value and Fraction and Percent Equivalents

Teacher Notes 4.2

Objectives

- Develop understanding of *decimal place value* using models.
- Identify decimal and percent equivalents for benchmark fractions.
- Identify decimal and percent equivalents for *proper fractions* and explain why they represent the same value.
- Identify decimal and percent equivalents for *mixed numbers* and explain why they represent the same value.

Vocabulary

Decimal
Place value (2.1)
Fraction (4.1)
Equivalent
Decimal point
Integer (3.1)
Mixed number
Power (1.1)
Denominator (4.1)
Benchmark fraction (4.1)
Numerator (4.1)

Prerequisites

Modeling, reading, and writing fractions and percents

Understanding concepts of place value

Get Started

- Draw a large box on the chalkboard or overhead projector.
- One by one, ask the students to come to the front of the room and write an expression inside the box that is equal to 100. Start the activity by writing 25 + 25 + 25 + 25. **Possible expressions: 10^2, 101 – 1, 10 × 10, 50 + 50, etc.**
- After each student has had a turn, write the phrase *Expressions Equivalent to 100* over the box. Tell students that there are always many ways to express a number. When describing the quantity 100, using the number 100 is the simplest way, but each expression in the box is *equivalent* to 100, and each is useful in certain applications.
- Tell students that there are several different ways to describe *part* of a number. They have already seen how to use fractions, ratios, and percents.

- Make another box and write the fraction $\frac{1}{2}$ inside the box. Label the box *Expressions Equivalent to* $\frac{1}{2}$. Ask students whether they have any ideas about other ways to express the quantity indicated by the fraction. Encourage them to give the percent and decimal equivalents. **Possible answers: 1:2, 50%, 0.50, $\frac{1}{4}+\frac{1}{4}$, etc.**
- Tell students that every fraction can be written as an equivalent decimal and an equivalent percent. This lesson will show them how to find the equivalents.

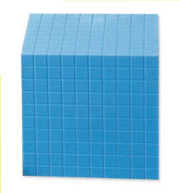
Subtopic 1 Decimal Place Value and Fraction Equivalents

Expand Their Horizons

This subtopic begins by teaching students how to read and to write decimal numbers. Students are more likely to internalize the concept of decimal place value if they are encouraged to read decimal numbers correctly. For example, reading 0.4 and "zero and four tenths" (or even "four tenths") provides a verbal cue to the meaning of the decimal number; the more casual "point four" provides no such cue. To test their understanding, try dictating a variety of decimal numbers and asking students to write their numerical equivalent.

Reading decimal numbers correctly will also help greatly when converting between decimal and fractional forms of numbers. For example, when the numbers $\frac{3}{10}$ and 0.3 are both read as "three tenths," it is easier to understand that they represent the same quantity.

To read a decimal number, students should read the number after the decimal point as if it were a whole number and then name the right-most decimal place value. So, 7.18 is read "seven and eighteen hundredths." Point out that the word "and" is only used in reading numbers to give the location of the decimal point. Reading 359 as "three hundred and fifty-nine" is incorrect.



In this section, only tenths and hundredths are modeled. Curious students might ask how thousandths could be modeled. If available, show students a thousandths-cube from a set of base-ten blocks and use it to show them how numbers like 0.005 might be modeled.

This section also introduces the concept of a mixed number. A mixed number is made up of an integer part and a fraction part and represents a quantity greater than one (when the mixed number is positive). Some students may connect mixed numbers to improper fractions, which can also represent quantities greater than one. Tell students that every mixed number has an improper fraction equivalent. These equivalents will be studied in Lesson 4.5.

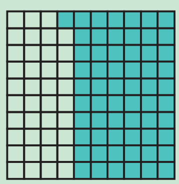
Some students may wonder about the use of the term *equivalent* instead of *equal*. Tell students that *equivalent* is usually reserved for numbers or expressions that represent the same quantity or that can be simplified to the same number.

1 There are 26 shaded squares out of 100 squares. Therefore, the decimal is 0.26, and the fraction is $\frac{26}{100}$. Point out that both expressions are read “twenty-six hundredths.”

2 The model shows a quantity greater than one; therefore, a mixed number is needed. There are two whole squares shaded, so the integer part of the mixed number is two. There are 33 out of 100 squares shaded in the third square, so the fraction part is $\frac{33}{100}$. The mixed number is $2\frac{33}{100}$. The decimal is 2.33. Point out that both expressions are read “two and thirty-three hundredths.”

Additional Examples

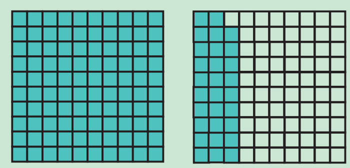
1. **Name the decimal and fraction shown by the shaded region.**



To find the decimal, count the number of shaded squares. Place the number after the decimal point so that the right-most digit lies in the hundredths place.

0.61 $\frac{61}{100}$

2. **Name the decimal and fraction shown by the shaded region.**



Form a mixed number using one as the integer part. Find the fraction part by counting the number of shaded squares and writing a fraction using 100 in the denominator.

1.29 $1\frac{29}{100}$

Subtopic 2 Changing Decimals to Fractions and Fractions to Decimals

Expand Their Horizons

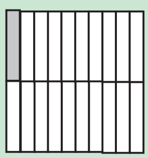
This subtopic introduces the idea of fraction-decimal equivalence. Any number that can be written as a fraction can also be written as an equivalent decimal. One way to find a decimal equivalent is to divide. For example, the fraction $\frac{4}{10}$ means $4 \div 10$. The quotient 0.4 is the decimal equivalent of $\frac{4}{10}$. Another more intuitive way to convert certain fractions to decimals is to use models. In this section, the denominator of each fraction is a factor of a power of 10. The model of each fraction can be converted into a decimal model by dividing the whole into 10 or 100 equal parts. (Fractions, for which the denominator is not a factor of a power of 10, like $\frac{1}{3}$, will be studied later in this course).

3 A model in which three of four equal parts are shaded can be transformed into a model with 75 of 100 equal parts shaded, showing that $\frac{3}{4}$ is equivalent to $\frac{75}{100}$, which is equivalent to 0.75. This may be a good time to point out the use of the Transitive Property in this example:
$\frac{3}{4} = \frac{75}{100}$, and $\frac{75}{100} = 0.75$, so $\frac{3}{4} = 0.75$.

4 The model of $\frac{1}{5}$ is transformed into a model with two of 10 equal parts shaded, showing that $\frac{1}{5} = \frac{2}{10} = 0.2$.

Additional Examples

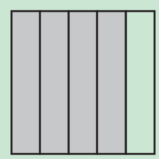
1. Find the decimal equivalent of $\frac{1}{20}$.



Draw vertical and horizontal lines as needed to transform the model into a hundredths-square. Count the number of shaded squares to find the decimal.

0.05

2. Find the decimal equivalent of $\frac{4}{5}$.



Draw vertical and horizontal lines as needed to transform the model into tenths or hundredths-square. Count the number of shaded squares to find the decimal.

0.8 or 0.80

Subtopic 3 Changing a Fraction to a Percent

Expand Their Horizons

In this subtopic, students find fraction and percent equivalents. To find the percent equivalent of a certain fraction, a model of the fraction is transformed into a hundredths model. Only fractions in which the denominator is 100 or a factor of 100 are studied. Percent equivalents of other fractions are studied later in the course.

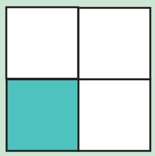
5 The fraction $\frac{57}{100}$ expresses a part-to-whole ratio in which the whole is 100. The numerator, 57, is the same as the percent. The percent equivalent of $\frac{57}{100}$ is 57%.

6 To find the percent equivalent of $\frac{3}{4}$, transform the model by drawing on horizontal and vertical lines to make a hundredths-square. Since 75 squares are shaded on the hundredths-square, $\frac{3}{4}$ = 75%.

7 Students should divide each fifth vertically in half, forming tenths. Then, draw horizontal lines to transform the tenths model into a hundredths model. Eighty out of 100 squares are shaded, so $\frac{4}{5}$ = 80%.

Additional Examples

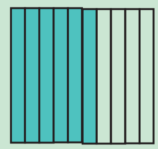
1. Find the percent equivalent of $\frac{1}{4}$.



Draw vertical and horizontal lines as needed to transform the model into a hundredths-square. Count the number of shaded squares to find the percent.

25%

2. Find the percent equivalent of $\frac{6}{10}$.



Draw vertical and horizontal lines as needed to transform the model into a hundredths-square. Count the number of shaded squares to find the percent.

60%

Subtopic 4 Benchmark Fractions and Fraction Equivalents

Expand Their Horizons

So far in this lesson, students have been shown models to help them visualize the equivalence of fractions, decimals, and percents. In this subtopic, it is suggested that they commit the three forms of certain benchmark fractions to memory. Encourage students who are having trouble with this section to visualize the models. Also, remind students that they really only have to memorize a few facts and can develop the other facts from them. For example, knowing that $\frac{1}{10}$ = 0.10 = 10%, students can use multiplication to figure out that $\frac{3}{10}$ = 0.30 = 30%; knowing that $\frac{1}{4}$ = 0.25 = 25%, they can use multiplication to figure out that $\frac{3}{4}$ = 0.75 = 75%; and so on.

8 Remind students that the decimal 0.6 is read “six tenths.” This may help get them started by providing a clue to the fraction equivalent, $\frac{6}{10}$.

$0.6 = 0.60 = \frac{6}{10} = \frac{60}{100} = 60\%$

9 Remind students that 40% means "40 out of 100." This clue may help them write the fraction equivalent: $\frac{40}{100}$. 40% = $\frac{40}{100}$ = 0.40 = 0.4

Additional Examples

1. **Give the fraction and percent equivalents of 0.9.**

   Use the fact that 0.1 = $\frac{1}{10}$ = 10%.

   $\frac{9}{10}$ $\frac{90}{100}$ **90%**

2. **Give the fraction and decimal equivalents of 10%.**

   10% means "10 out of 100."

   $\frac{10}{100}$ **0.10 or 0.1**

Look Beyond

In this lesson, it was stated that every fraction can be written as a decimal. The decimal equivalents seen in this lesson are *terminating decimals*. That is, they have a finite number of digits after the decimal point. Some fractions, however, are equivalent to *repeating decimals*. These decimals have an infinite number of digits after the decimal point. These digits repeat in a pattern. Students will study repeating decimals in a future lesson. For now, they may be intrigued by finding repeating decimal equivalents of fractions on a calculator. Ask them to find the decimal equivalents of $\frac{1}{4}$, $\frac{1}{12}$, $\frac{13}{99}$, and $\frac{3}{111}$ by using division. Ask them to determine which fractions repeat and which terminate. Ask them to identify the repeating digits in the repeating decimals.

Connections

Students may wonder why it is worthwhile to be able to change between fraction, decimal, and percent representations of a number. Tell them that different forms of the number are used in different applications. For example, when finding the cost of a sweater normally priced at $40 but on sale for 30% off, we use the percent form to express the portion of the price that is discounted. We use the decimal form of 30%, or 0.30, in the computation to find the discount: 0.30 × $40 = $12.