Numbers and Operations

★ Module 4 ★

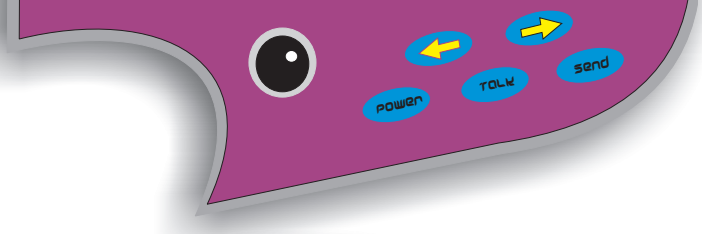
Fractions, Decimals, Percents, and Factors

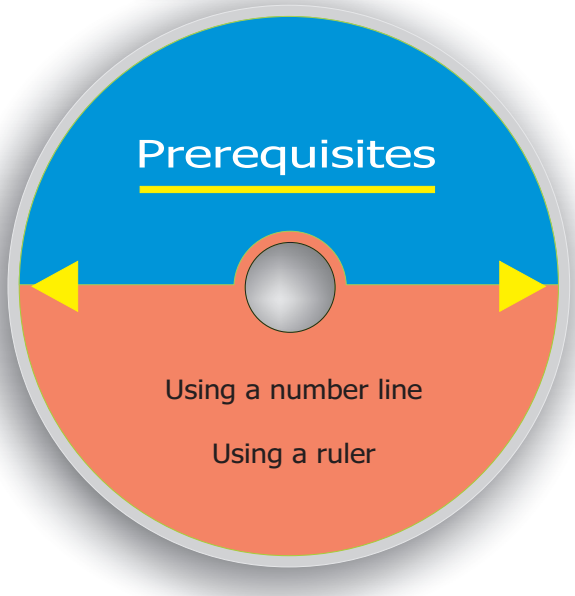
Lesson 1
Concepts of Fractions, Ratios, and Percents

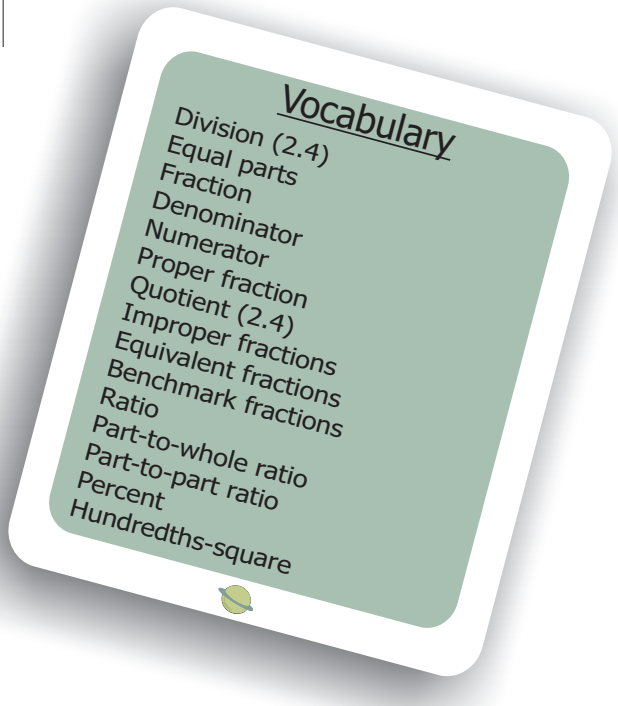
Teacher Notes 4.1

Objectives

- Use models and visual representations to develop the concepts of the following *fractions*: parts of unit wholes, parts of a collection, locations on number lines, locations on rulers (*benchmark fractions*), division of whole numbers.
- Use models and visual representations to develop the concepts of the following *ratios*: part-to-part (two boys to three girls) and part-to-whole (two boys to five people).
- Use models and visual representations to develop the concepts of *percents*: part-to-100.







Get Started

- Ask students to stand up from their desks. Move the boys to one side of the room and the girls to the other. (Alternatively, divide the students into two groups by grade level, first letter of last name, or some other distinction.)
- Explain to students that the objective is to find a way to describe the portion of the class that is made up of girls. Be sure students understand they are to describe the portion of the class that is girls, not the actual number of girls in the class. Ask the students for suggestions. Possible suggestions include fraction, percent, decimal, and ratio.
- Count the number of girls. Ask the class to consider why anyone might be interested in describing the *part* of the class that is girls rather than the *actual number* of girls. For example, why might it be helpful to know that the fraction of girls is $\frac{12}{19}$ rather than simply knowing there are 12 girls in the class? Possible answer: Knowing the fraction gives a better idea of how the class is composed.

Write the headings *number of boys*, *number of girls*, and *number of students* on the board with the appropriate numbers underneath. Tell students that in this lesson they will learn several different ways to use these numbers to describe the relationships among the three quantities.

Subtopic 1 Fractions

Expand Their Horizons

In this subtopic, fractions are defined. Emphasize that the denominator of a fraction specifies the number of *equal parts* into which a unit whole is divided. The numerator specifies the number of equal parts being considered. Emphasize the idea of equal parts by showing students the following diagram:

Ask students to find the error in the following logic: *There are three parts. One is shaded. The fraction of the figure that is shaded is* $\frac{1}{3}$. Students should see that the three parts are not *equal parts;* therefore, the logic is invalid.

Be sure students understand that a fraction shows a division problem. Students should practice writing simple division facts, such as 42 ÷ 6 = 7, using fraction notation: $\frac{42}{6} = 7$. Consistently reminding them that a fraction bar can be read as a division symbol will help students simplify improper fractions in the future lessons.

When a fraction represents a quantity that is greater than or equal to zero but less than one, it is called a *proper fraction.* When plotted on a number line, a proper fraction lies to the left of one. When a fraction represents a quantity greater than or equal to one, it is called an *improper fraction,* and is plotted at one or to the right of one on a number line.

When representing fractions on a ruler, visual and tactile learners may benefit from looking at and holding an actual ruler, if available. Ask students to look at the ruler and determine the number of equal parts in one whole inch (most rulers are divided into sixteenths). Show them how the space between each hash mark represents one sixteenth of an inch and ask them to measure some small objects in the classroom (length of a pencil lead, width of a penny, etc).

The term *benchmark fraction* may be new to students. Benchmark fractions are fractions of which students should have an immediate mental image. These fractions are most commonly used in everyday circumstances; other more complicated fractions are often rounded and simplified to benchmark fractions in the media. For example, people are more likely to hear: "*Nearly* $\frac{1}{3}$ *of students play sports,"* rather than "*351 out of 1,108 students play sports." T*hey can form an immediate mental image of $\frac{1}{3}$, while $\frac{351}{1,108}$ is more complicated to understand.

1 There are three equal parts. Two are shaded. The fraction is $\frac{2}{3}$.

2 There are five identical moons. Three of them are blue. The fraction is $\frac{3}{5}$.

Common Error Alert:

Students may incorrectly use the number of unshaded moons as the denominator. They may write the fraction $\frac{3}{2}$ instead of $\frac{3}{5}$. Remind them that the denominator gives the total number of moons.

3 The space from zero to one is divided into four equal parts. The point is one part to the right of zero. The fraction is $\frac{1}{4}$.

4 The ruler is divided into eight equal parts. The rectangle is three parts long. So, the fraction is $\frac{3}{8}$.

Common Error Alert:

Students may confuse the number of hash marks with the number of equal parts. In this example, they may count five hash marks and write the incorrect fraction $\frac{1}{5}$. Remind them to count the number of equal parts, or the number of "jumps" from zero to one, not the number of hash marks.

5 Remind students that dividing the space between zero and one into *x* equal parts means they should draw *x* – 1 hash marks between zero and one. To divide the space into two equal parts, they should draw one hash mark between zero and one. To divide the space into three equal parts, they should draw two hash marks between zero and one, and so on.

Additional Examples

1. Name the fraction shown by the shaded region.

Write a fraction using the number of shaded parts as the numerator and the total number of equal parts as the denominator.

$\frac{5}{6}$

2. Name the fraction of triangles that is shaded.

Write a fraction using the number of shaded triangles as the numerator and the total number of triangles as the denominator.

$\frac{2}{7}$

Subtopic 2 Ratios: Part-to-Whole

Expand Their Horizons

This subtopic introduces the concept of ratio. In this section, only part-to-whole ratios are considered. A fraction is one way to express the ratio of a part to the whole.

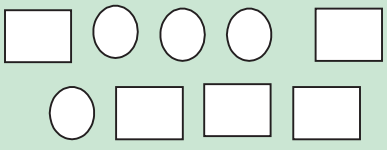
Ratios can be expressed using any one of several acceptable notations. Encourage students to use each notation occasionally.

6 There are four circles and nine shapes. The ratio of circles to shapes is 4 to 9. Encourage students to see the relationship between fraction and ratio in this example. The fraction of the shapes which are circles is $\frac{4}{9}$.

7 There are three girls and five people. The ratio of girls to people is 3 to 5.

Additional Examples

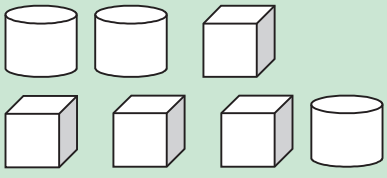
1. What is the ratio of rectangles to all shapes?



Count the number of rectangles. Count the the number of shapes. Write a ratio using the using the word “to,” a colon, and a fraction.

5 to 9 5:9 $\frac{5}{9}$

2. What is the ratio of cubes to all figures?



Count the number of cubes. Count number of figures. Write a ratio word “to,” a colon, and a fraction.

4 to 7 4:7 $\frac{4}{7}$

Expand Their Horizons

This subtopic introduces part-to-part ratios. This ratio gives an immediate comparison of parts within a whole. For example, the ratio of cats to dogs, 3:5, immediately gives the listener an idea of how the numbers of each animal compare. Ask students whether they have heard ratios used in everyday applications. For instance, some students may have heard of *student-teacher ratios*. Other students may have heard of *aspect ratios* on video equipment, which compares the length of a video screen to its width.

8 There are four circles and five squares. The part-to-part ratio is 4:5.

Common Error Alert:

Students may confuse the order of the numbers when writing a ratio. In this exercise, they may incorrectly write the ratio as 5:4. Remind them that the order of the numbers is very important. They should develop the habit of checking their ratios against the words in the problem to be sure they have correctly arranged the numbers in the ratio. If students are having difficulty, have them label the units in their ratios. Writing "4 circles: 5 squares" or "2 boys: 3 girls" will help eliminate any confusion. Until students have a better understanding of ratios, they may need to label all answers.

9 There are two boys and three girls. So, the ratio of boys to girls is 2 to 3.

Additional Examples

1. What is the ratio of circles to hearts?

Count the number of circles. Count the number of hearts. Write a ratio using the word "to," a colon, and a fraction.

3 to 5 3:5 $\frac{3}{5}$

2. What is the ratio of moons to suns?

Count the number of moons. Count the number of suns. Write a ratio using the word "to," a colon, and a fraction.

3 to 4 3:4 $\frac{3}{4}$

Expand Their Horizons

In this subtopic, students are introduced to a special ratio: percent. A percent is a ratio comparing a part to 100; in a percent, the "whole" is always 100. To model percents, a hundredths-square is used.

The purpose of this section is to give students a visual understanding of percents and to relate percents to fractions and ratios. Students will study more complex percent-decimal-fraction equivalents within this module. Applications and problem solving with percents will be presented in Module 7.

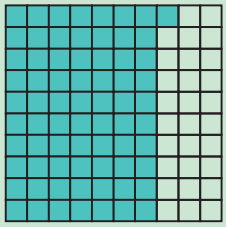
Before viewing the lesson, it may be helpful to introduce the hundredths-square to students and demonstrate that the hundredths-square is made up of 10 columns containing 10 squares each. Each small square represents one hundredth.

10 Eighty-two of the 100 squares on the hundredths-square are shaded. The percent shaded is 82%.

11 In this exercise, students are asked to represent the portion of squares that are shaded in three different ways: as a fraction, as a ratio of shaded squares to total number of squares, and as a percent. Be sure students understand that these are three different ways to express the same relationship. There are 63 squares shaded, so the fraction is $\frac{63}{100}$; the ratio is 63:100; and the percent is 63%.

Additional Examples

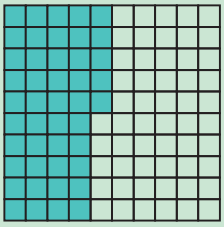
1. What percent of the squares is shaded?



Count the number of shaded squares. There are 100 squares altogether, so the number of shaded squares is the same as the percent.

71%

2. What fraction is shaded? What is the ratio of shaded squares to total squares? What percent of the squares is shaded?



Count the number of shaded squares. There are 100 squares altogether. Use 100 as the denominator of the fraction and as the "whole" in the ratio.

$\frac{45}{100}$ 45:100 45%

Look Beyond

Students will continue to study and use fractions, ratios, and percents throughout this course and future courses. For example, in an algebra course, they will write fractions in which the numerator and/or denominator contain variables. These fractions are called *rational expressions*. They will learn how to find the percent of a number, how to find percent increase and decrease, and how to use percent equivalents to simplify expressions and to solve problems.

Connections

Fractions, ratios, and percents are found commonly in everyday applications. To help students see their prevalence, ask them to use a section of newspaper to determine which form of the fraction-ratio-percent relationship seems to be used most often. Can they find instances of benchmark fractions used to describe more complicated fractions? Can they find instances of ratio? What kind of information is communicated using percents? Students may also work in groups and present their findings to the class.