Numbers and Operations

Module 3

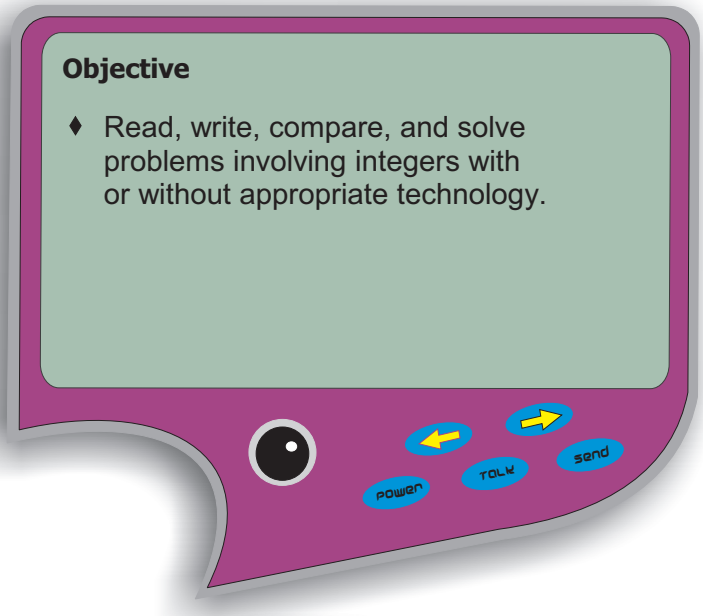
Integers

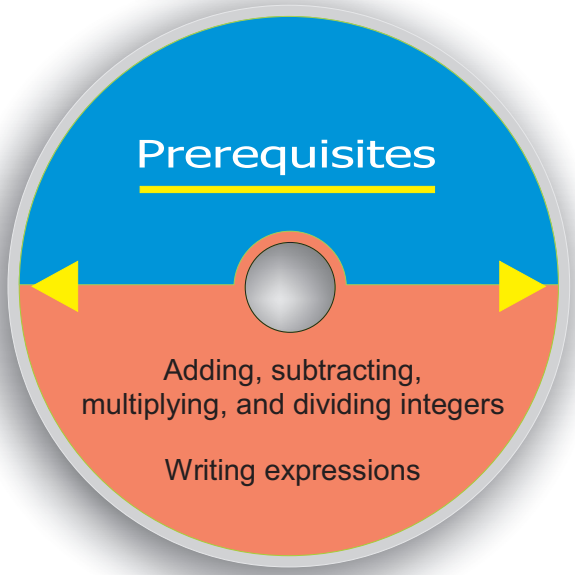
Lesson 5
Solving Problems with Integers

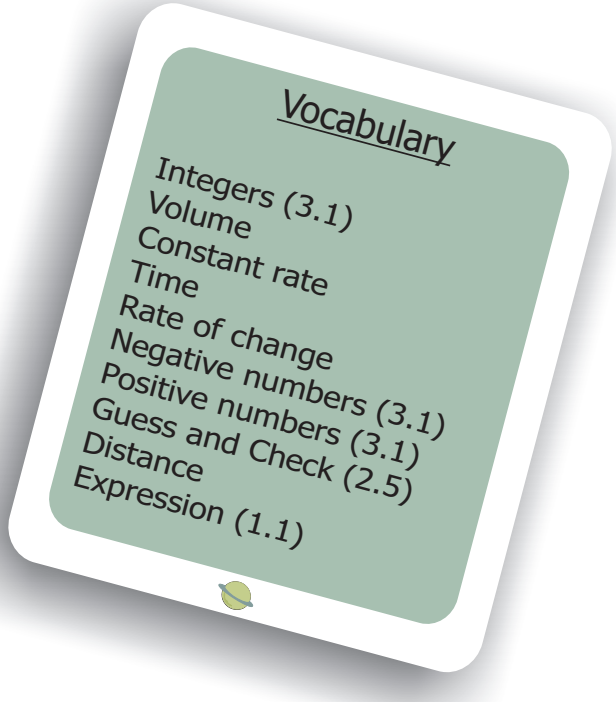
Teacher Notes 3.5

Objective

- Read, write, compare, and solve problems involving integers with or without appropriate technology.







Get Started

- Present the following scenario to students: *Rory had $40 in her bank account. Then, she made three $10 deposits*. Ask students to write and evaluate an expression to find Rory's final balance.
  $40 + $10 + $10 + $10, or $40 + 3($10) = $70
- Describe another bank customer for whom the expression is $100 + 4(–$5). Ask students to identify what each number in the expression could represent.
  Possible answer: 100 represents the beginning balance, four represents the number of transactions, and negative five represents the amount of the transaction. Ask students to explain what a transaction amount of negative five dollars means. **a $5 withdrawal**
- Ask students to evaluate the expression $100 + 4(-$5). Call on a volunteer to explain the steps needed to simplify the expression.
  $100 + 4(-$5) = $100 + (-$20) = $80. Ask the students what the 80 represents.
  an ending balance

- Tell students that another bank customer had a similar situation and that the expression representing his situation was -$50 + 6($80). Ask students to translate the expression from symbols to words. **Possible answer: The customer was overdrawn by $50 and then made six equal deposits of $80 each.** Ask students to evaluate the expression. Call on a volunteer to explain the steps needed to simplify the expression.
  -$50 + 6($80) = -$50 + $480 = $430
- Tell students that in this lesson they will write and evaluate expressions. They will evaluate the expressions using the rules for operations on integers as well as the rules of order of operations.

Subtopic 1 Solving Problems with Integers 1 (Math Camp Swimming Pool)

Expand Their Horizons

In Subtopic 1, students solve word problems involving integers. Each word problem uses the scenario of draining and/or filling the math camp swimming pool.

In the problems, the expression

given amount + (rate of change)(time of change)

is used to determine a future or past volume of water in the pool. Tell students that "given amount" means the amount currently in the pool.

Be sure students understand that the rate of change of the water level is negative when water is draining and positive when the pool is being filled. Also, be sure they understand when to make the measurement of time positive or negative. Tell them to look for key words and phrases to indicate whether to use a positive or negative integer. Times in the future require a positive integer; times in the past require a negative integer. Key words and phrases like "five hours from now" (which indicates the future) and "four hours ago" (which indicates the past) can help them decide.

When simplifying expressions like 24 + (+2)(-4), remind students that order of operations applies to operations with integers. So, they must complete the multiplication first, and then add.

A negative integer is used to represent time, and a positive integer is used to represent the rate of change.
21 + (+2)(-6) = 21 + (-12) = 21 – 12 = 9.
There were nine gallons of water in the pool six hours ago.

A negative integer is used to represent time, and a negative integer is used to represent the rate of change.
7 + (-3)(-6) = 7 + (+18) = 7 + 18 = 25.
There were 25 gallons of water in the pool six hours ago.

3 In previous problems, the starting volume, time, and rate of change were known. The final volume of water in the pool had to be found. In this problem, the unknown quantity is time. The final volume of water in the pool is zero and the starting volume and rate of change are known. The characters in the DVD use the *Guess and Check* method to find the time, but many students may be able to tell by looking that the time must be six hours in order for the volume to equal zero.

Finding the time for which 12,000 + (-2000)(time) is equal to zero is tantamount to solving the equation 0 = 12,000 + (-2000)*t*. Advanced students may benefit from seeing the equation and the steps needed to solve it.

4 In this problem, the starting volume, final volume, and rate of change are known. The unknown quantity is time. Students must find the time for which 0 + (1,200)(time) is equal to 12,000. They can use the *Guess and Check* method and number sense to find that it will take 10 hours.

Additional Examples

1. The space camp pool currently has 60 gallons of water. It is being filled at a rate of eight gallons per hour. What was the volume of water in the pool four hours ago?

Substitute values from the problem into the expression and evaluate.

current amount + (rate) (time)
60 + (+8)(-4)
60 + (-32)
60 – 32
28

There were 28 gallons of water in the pool four hours ago.

2. The space camp pool currently has 14,000 gallons of water. It is being drained at a rate of 700 gallons per hour. How many hours will it take to drain the pool?

The unknown quantity is time. Use *Guess and Check* or number sense to find the value of time for which the expression is equal to zero.

current amount + (rate) (time)
14,000 + (-700)(time)

Guess:
10 hours: 14,000 + (-700)(10) = 7,000

Guess: 20 hours: 14,000 + (-700)(20) = 0

It will take 20 hours to drain the pool.

Subtopic 2 Solving Problems with Integers 2 (Zeo's Alienoon)

Expand Their Horizons

In Subtopic 2, students solve integer problems about an "alienoon" (hot air balloon) traveling up and down along an imaginary number line. Be sure students understand that movement upward is represented by a positive number and movement downward is represented by a negative number. Also, emphasize that the position "0" lies somewhere *above* the ground, on a platform.

Remind students that the end result of successive moves of the alienoon can be found by adding the expressions for (rate)(time) for each move. So, the end result of starting at zero, ascending at three units/hour for two hours, then descending at four units/hr for two hours is (3)(2) + (-4)(2).

5 The alienoon is descending, so the rate of change is negative three. The key word "ago" indicates that a negative number should represent time. (-3)(-4) = 12. The balloon was at 12 units four hours ago.

6 Write products to represent the descent and subsequent ascent and then add the expressions. (-3)(3) + (2)(2) = -9 + 4 = -5

Additional Examples

1. The alienoon is four units above the platform. It is descending at three units/hour. Where was the alienoon two hours ago?

Describe the rate of descent using a negative integer. Describe time using a negative integer.

$$4 + (-3)(-2)$$
$$4 + 6$$
$$10$$

The alienoon was 10 units above the platform two hours ago.

2. The alienoon starts at zero and then ascends at three units/hour for two hours. Next, it descends at four units/hour for three hours. Where is it?

Write products to represent the ascent and descent. Then, add the expressions.

$$(3)(2) + (-4)(3)$$
$$6 + -12$$
$$-6$$

The alienoon is six units below the platform.

Look Beyond

Many of the examples in this lesson featured a starting value followed by the product of a rate of change and a variable (time). These expressions can be written in the form $mx + b$, where m is the rate of change, x is the variable, and b is the "starting value." Equations written as $y = mx + b$, where y is the "resulting value," are called *linear equations* and are studied extensively in algebra courses.

Calculator

For enrichment, show students how to evaluate expressions such as 24 + (2)(-4) on standard, scientific, and/or graphing calculators. Remind them to determine whether or not their calculator performs operations in the order entered or whether it obeys the order of operations.