Numbers and Operations

Module 3

Integers

Lesson 4
Multiplying and Dividing Integers

Teacher Notes 3.4

Objectives

- Model multiplication and division of integers using physical objects and pictures.
- Multiply integers.
- Divide integers.

Prerequisites

Using two-color counters

Multiplying and dividing whole numbers

Using a number line

Vocabulary

Factors (1.2)
Integer (3.1)
Product (2.3)
Quotient (2.4)
Zero pair (3.2)
Positive numbers (3.1)
Negative numbers (3.1)

Get Started

- As an introduction to the modeling of multiplication shown in this lesson, give each student several positive and negative counters and a blank piece of paper to serve as a workspace.
- Ask students how they can model the sum 2 + 2 + 2 using counters. **Place counters on the workspace in three groups of two. Then find the total number of counters, six.**
- Remind students that multiplication is repeated addition. Ask them to write an expression that shows 2 + 2 + 2 using multiplication. **3 × 2** (Be sure students write 3 × 2 rather than 2 × 3, since 3 × 2 means “three groups of two” and 2 × 3 means “two groups of three.”) On the board, write 3 × 2 = 6.

- Next, ask students to model -2 + -2 + -2 using counters. (Workspaces should show six negative counters in three groups of two.) Again, ask students to use multiplication to model the repeated addition of negative two. On the board, write 3 × -2 = -6.
- Point to the two equations written on the board. Point out that repeatedly adding a positive number results in a positive number, as in 3 × 2 = 6. Then, point out that repeatedly adding a negative number results in a negative number, as in 3 × -2 = -6.
- Finally, ask students to consider the product -2 × -3. Ask them to speculate how they might form "negative two groups of negative three." Tell them that the model will have to become more sophisticated and that this lesson will show them several ways to model and find the product of two factors when the first factor is a negative number.

Subtopic 1 — Multiply Integers Using Counters

Expand Their Horizons

In Subtopic 1, students multiply integers using counters. The method for modeling integer multiplication using counters may be confusing for students. It may be helpful to provide a display of the rules to which students can refer when looking at or creating models.

first factor	×	**second factor**
number of groups		number in each group

*If first factor is **positive**, put counters **on**.*
*If first factor is **negative**, take counters **off**.*

Help students understand the models by having them practice translating products from number form into word form. For example, have them translate the product -4 × 6 into "take off four groups of positive six" and the product 5 × -2 into "put on five groups of negative two."

1 Model -3 × 1 by taking off three groups of one positive counter each. Since the workspace is empty, three zero pairs must be placed on the workspace before three positive counters can be removed. The result is three negative counters, so -3 × 1 = -3.

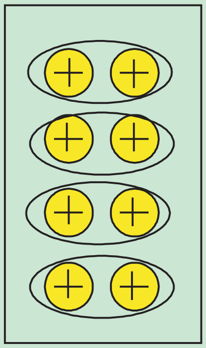
2 Model 4 × -2 by putting on four groups of two negative counters. The result is eight negative counters, so 4 × -2 = -8.

3 Model -3 × -4 by taking off three groups of four negative counters. Since the workspace is empty, three groups of four zero pairs must be placed on the workspace before three groups of four negative counters can be removed. The result is 12 positive counters, so -
-3 × -4 = 12.

Additional Examples

1. Use counters to multiply.
4 × 2

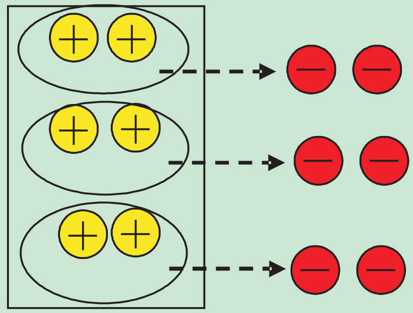
Place four groups of two positive counters on the workspace.



4 × 2 = 8

2. Use counters to multiply.
-3 × -2

Take off three groups of two negative counters, or six negative counters. Since the workspace is empty, add six zero pairs so that six negative counters can be removed.



-3 × -2 = 6

Subtopic 2 — Multiply Integers Using a Number Line

Expand Their Horizons

In Subtopic 2, students are shown how to model multiplication of integers on a number line. As with the previous model, students may become confused as to how to model products. Post the following rules in the classroom for their reference.

first factor	**x**	**second factor**
direction to face		length of each stop
number of steps to take		direction to walk (forward or backward)
ALWAYS START AT 0!		

Be sure students understand the distinction between “direction to face” and “direction to walk.” Point out that the “negative direction” means facing left on the number line, while the “positive direction” means facing right.

Kinesthetic learners may benefit from acting out the model on a life-size scale. Set up a number line on the classroom floor using masking tape. Then, have students model products such as 5 × -1, -2 × -4, and 3 × 4.

Help students understand the number line model by practicing modeling products. For example, have them model 4 × 2 by starting at zero, facing in the positive direction, and taking four two-unit steps forward. Have them model -3 × -5 by starting at zero, facing in the negative direction and by taking three five-unit steps backwards.

4 To model -2 × 5, start at zero and face the negative direction. Then, take two five-unit steps forward. The result is -10. So, -2 × 5 = -10.

5 To model 3 × -5, start at zero and face the positive direction. Then, take three five-unit steps backward. The result is -15. So, 3 × -5 = -15.

6 To model -3 × -5, start at zero and face the negative direction. Then, take three five-unit steps backward. The result is 15. So, -3 × -5 = 15.

Point out the similarities and differences between Lesson Notes Problems 5 and 6. The absolute values of the factors are the same, as are the absolute values of the products. However, in Problem 5, the factors have different signs, and the product is negative; in Problem 6, the factors have the same sign, and the product is positive.

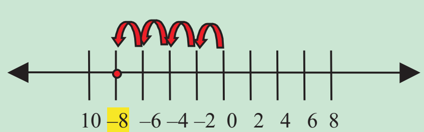
Common Error Alert:

If students become frustrated or confused by the models, encourage them to use the models and to look for patterns in the results of modeling products. Reassure them that the patterns will help them establish and remember the rules for multiplying integers.

Additional Examples

1. Use a number line to multiply.
4 × -2

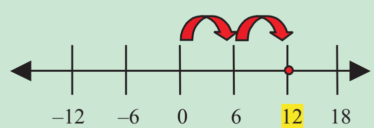
Start at zero. Face the positive direction; then take four two-unit steps backward.



4 × -2 = -8

2. Use a number line to multiply.
-2 × -6

Start at zero. Face the negative direction; then take two six-unit steps backward.



-2 × -6 = 12

Expand Their Horizons

In Subtopic 3, rules for multiplying and dividing integers are given. Before viewing the DVD, ask students to look back at the results of modeling products. Have them write equations (e.g. -3 × -5 = 15) for each product. Ask them to use the results to make a conjecture about the rules for multiplying integers. As a hint, tell them that the sign of the product depends on the signs of the factors.

7 The dividend and divisor have different signs. The quotient will be negative. Since 72 ÷ 9 = 8, -72 ÷ 9 = -8.

8 The factors have different signs. The product will be negative. Since 25 × 4 = 100, 25 × -4 = -100.

9 The dividend and divisor have the same sign. The quotient will be positive. Since 35 ÷ 7 = 5, -35 ÷ -7 = 5.

10 The factors have the same sign. The product will be positive. Since 6 × 8 = 48, -6 × -8 = 48.

Additional Examples

1. Multiply.
5 × -8

The factors have different signs. The product will be negative.

5 × -8 = -40

2. Divide.
-45 ÷ -9

The dividend and divisor have the same sign. The quotient will be positive.

-45 ÷ -9 = 5

Look Beyond

The rules for multiplying and dividing integers apply to all real numbers. Students will be applying the rules to fractions and decimals as well as integers. For example, the rules will be used to evaluate expressions such as -1.5 × -4 and 8 ÷ (-¼).

Challenge advanced students to consider what happens when more than two numbers are multiplied or divided. For example, ask them to evaluate -2 × -3 × -5.

Connections

The phenomenon by which two negatives make a positive is not exclusive to arithmetic. From their language arts courses, students may be familiar with the term "double negative." A double negative, which is considered nonstandard English, occurs when two negative words (or other forms of negation) are used in the same sentence. The effect is that the two negatives cancel each other out, leaving a positive result. For example, "She did *not* eat *no* ice cream," effectively means "She *did* eat *some* ice cream." "The umpire's call was *not un*fair," effectively means "The umpire's call *was fair*."