Numbers and Operations

Module 3

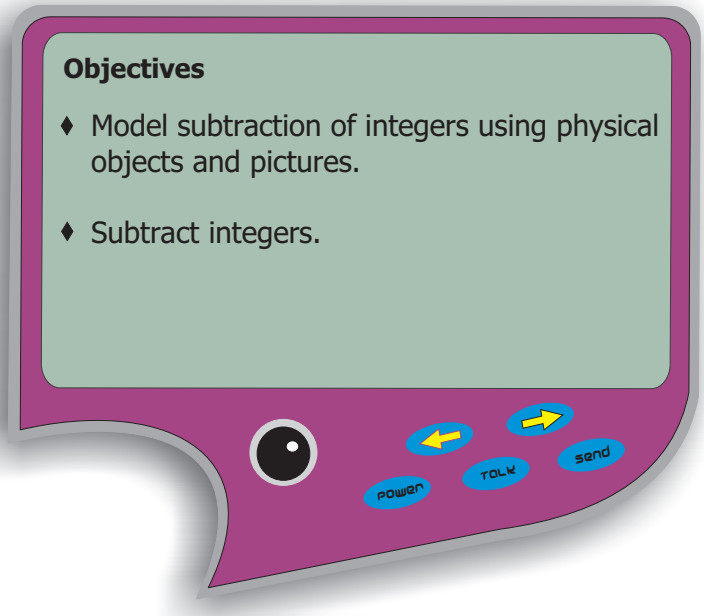
Integers

Lesson 3
Subtracting Integers

Teacher Notes 3.3

Objectives

- Model subtraction of integers using physical objects and pictures.
- Subtract integers.



Prerequisites

Adding, subtracting, multiplying and dividing whole numbers

Defining, comparing and adding integers

Finding opposites

Vocabulary

Negative numbers (3.1)
Positive numbers (3.1)
Integers (3.1)
Opposites (3.1)
Absolute value (3.1)
Minuend (2.2)
Subtrahend (2.2)
Difference (2.2)

Get Started

- To introduce the lesson, ask students to review how to use manipulatives and number lines to model the subtraction of numbers.
- Give students counting manipulatives. Ask them to model subtraction expressions such as 8 – 2 and 4 – 3 using the manipulatives. Ask how they can use the counters to model the problem and find the difference. **Start with the first number of counters and then take away the second number of counters. The difference is the number of counters remaining.**
- Ask students to use a number line to model subtraction expressions such as 6 – 2 and 9 – 4. Ask them to verbalize a rule that describes how to model subtraction on a number line. **Start at the position of the first number and then move to the left the number of places given by the second number.**
- Tell students that in this lesson they will use counters and number lines to model the subtraction of integers. Ask students to speculate how modeling subtraction using counters and number lines might be different when negative numbers are used.

Expand Their Horizons

In Subtopic 1, students use counters to subtract integers. In many cases, the model of the minuend must be modified before the subtraction can be modeled. Be sure students understand that adding a zero pair to the minuend does not change its value. Emphasize this point by modeling the number two with counters. Add a zero pair and ask students to give the value of the counters (2). Continue adding and taking away zero pairs, each time asking students to give the value. Make sure they understand that the value is always two.

This subtopic uses the terms *minuend* (from the Latin "that which must be lessened") and *subtrahend* (from the Latin "that which must be taken away"). Encourage students to become comfortable with these terms by using them throughout the lesson.

1 Model the minuend using three red counters. The subtrahend requires that one yellow counter be taken away, so add one zero pair to the minuend. Take away one yellow counter. The result is four red counters, so -3 – 1 = -4.

Common Error Alert:

When subtracting integers, students may lose track of one or more negative signs. They may benefit from a visual cue. When presented with a new problem, suggest that they immediately circle the minuend and put a box around the subtrahend.

2 On the DVD, the character Luria generalizes that "subtracting a negative ... is like adding a positive." Point out this generalization and tell students that they will formalize rules for subtracting integers later in the DVD. To solve the problem, model the minuend using four yellow counters. The subtrahend requires that two red counters be taken away, so add two zero pairs to the minuend. Take away two red counters. The result is six yellow counters. So, 4 – (-2) = 6.

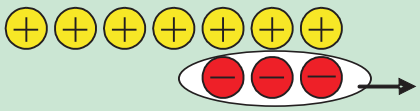
3 Model the minuend using three red counters. The subtrahend requires that four red counters be taken away, so add one zero pair to the minuend. Take away four red counters, leaving one yellow counter. -3 – (-4) = 1.

4 Model the minuend using three yellow counters. In order to be able to take away five yellow counters, add two zero pairs to the minuend. Take away five yellow counters, leaving two red counters. So, 3 – 5 = -2.

Additional Examples

1. Subtract.
4 – (-3)

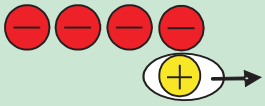
Model four using four yellow counters. Add three zero pairs to the minuend. Take away three red counters, leaving seven yellow counters.



4 – (-3) = 7

2. Subtract.
-3 – 1

Model negative three using three red counters. Add one zero pair to the minuend. Take away one yellow counter, leaving four red counters.



-3 – 1 = -4

Subtopic 2 Subtracting Integers Using Number Lines

Expand Their Horizons

In Subtopic 2, students subtract integers using a number line. Before showing how to use a number line to subtract, students are given the rule *to subtract an integer, add its opposite*. Before viewing this subtopic of the DVD, take a moment to show that subtraction is tantamount to adding the opposite. For example, ask students to find 10 – 4 and then ask them to find 10 + (-4) using number lines.

To help students understand that $a - b$ is equivalent to $a + (-b)$, ask them to consider real-life scenarios. For example, show how each of the expressions 5 – 2 and 5 + (-2) can represent the scenario *Ron had $5 and then he spent $2.* Show how each of the expressions -2 – 4 and -2 + (-4) can represent the scenario *Mary was two feet underwater; she then descended four more feet*.

5

To find -3 – (-4), write the equivalent expression -3 + 4. To add on a number line, start at negative three and move four places to the right. So, -3 – (-4) = 1.

Common Error Alert:

Students might write the equivalent addition expression incorrectly. They often change subtraction to addition but fail to change the sign of the subtrahend. Make sure they check their addition expressions carefully before adding.

6 To find 1 – 4, write the equivalent expression 1 + (-4). To add on a number line, start at one and then move four places left. So, 1 – 4 = -3.

7 To find -3 – 2, write the equivalent expression -3 + (-2). Model the addition on the number line by starting at negative three and moving two places to the left. -3 – 2 = -5.

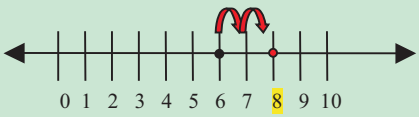
Common error alert:

Students may be tempted to skip the step of writing the equivalent addition expression. Encourage them to write the expression until they gain proficiency in subtracting integers.

Additional Examples

1. Subtract.
6 – (-2)

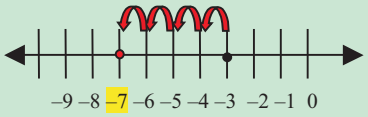
Write the equivalent addition expression 6 + 2. Start at 6; then move 2 units right.



6 – (-2) = 8

2. Subtract.
-3 – 4

Write the equivalent addition expression -3 + (-4). Start at -3; then move 4 units left.



-3 – 4 = -7

Subtopic 3 Subtracting Integers Using Rules

Expand Their Horizons

In Subtopic 3, students are given rules for subtracting integers. Because these rules rely on the rules for adding integers learned in the previous lesson, a review of those rules with additional exercises may be needed.

As subtraction expressions to be evaluated are presented, read the subtraction sign not only as “minus” but also as “plus the opposite of.” For example, present -6 – (-7) as “negative six minus negative seven” and then as “negative six plus the opposite of negative seven.” This auditory cue might help struggling students.

8 In the related addition expression, -11 + (-21), both addends have the same sign. So, the sum is negative. -11 – 21 = -11 + (-21) = -32.

9 To find 25 – (-14), write the related addition expression 25 + 14. Both addends are positive. So, 25 – (-14) = 25 + 14 = 39.

Additional Examples

1. Subtract.
-9 – (-1)

Write the related addition and then add.

-9 – (-1)
-9 + 1
-8

2. Subtract.
-16 – 5

Write the related addition and then add.

-16 – 5
-16 + (-5)
-21

Look Beyond

In this lesson and the previous lesson, students have seen how to add and to subtract integers. The rules learned in these lessons apply not only to integers, but also to all real numbers. Eventually, students will find sums and differences of signed fractions and decimals using these rules.

Point out to students that the study of integers provides a departure from several fundamental ideas they may have previously held about numbers and operations. In Lesson 1, they were taught that the number line extends to the left of zero. In Lesson 2, they were taught that addition does not always make a number bigger. In this lesson, they were taught that subtraction does not always make a number smaller. Call attention to the fact that their world of mathematics is expanding.

Connections

Subtracting negative numbers occurs in everyday situations in which the difference between a measurement *under* (or *before*) and a measurement *over* (or *after*) is found. For example, the difference in elevation between a ship's mast at 40 feet above sea level and its rudder at five feet below sea level can be found using the expression 40 – (-5). Or, the time elapsed between an event that occurred in 1200 A.D. and one that occurred in 300 B.C. can be found using the expression 1200 – (-300). In practical use, however, people often think of finding these differences using addition. That is, they add the distances from zero. So, the difference in elevation is found using the expression 40 + 5; the time elapsed is found using the expression 1200 + 300.