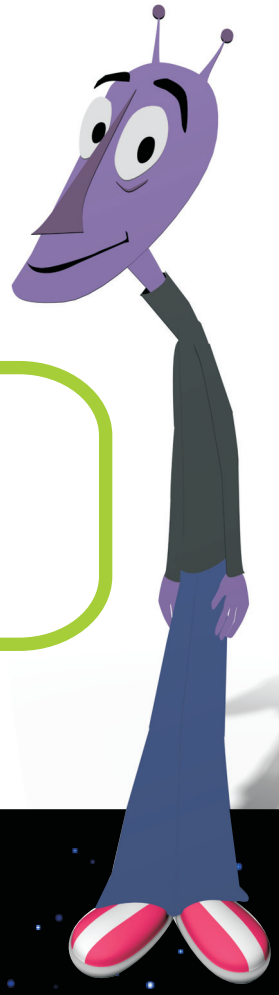
Numbers and Operations

Module 3

Integers

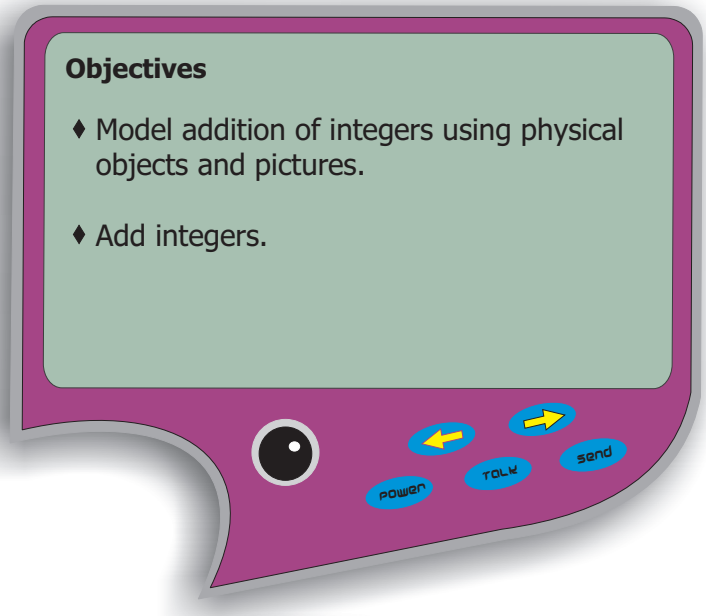


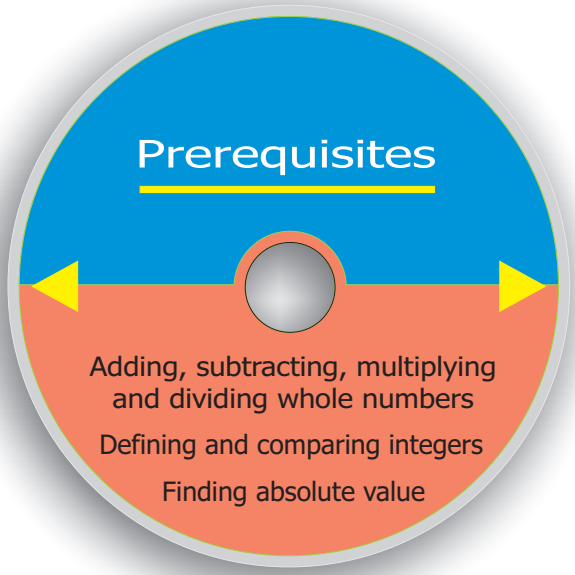
Lesson 2
Adding Integers

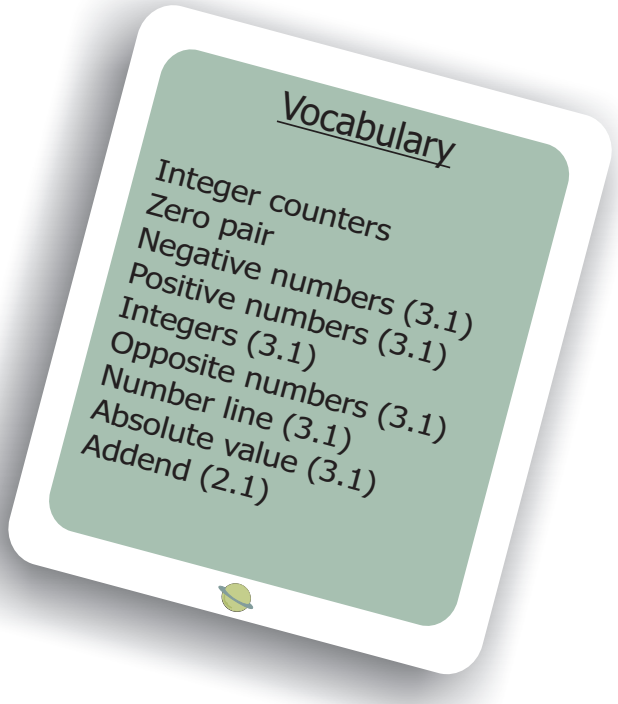
Teacher Notes 3.2

Objectives

- Model addition of integers using physical objects and pictures.
- Add integers.







Get Started

- Review how to use manipulatives and number lines to model addition.
- Give students counting manipulatives such as counters, beans, or small candies. Ask students to model addition expressions such as 2 + 3 and 1 + 6 using the manipulatives. Ask how they can use the counters to model each addend and to find the sum. **Each addend is represented by the corresponding number of counters; the sum is the total number of counters when all addends are modeled.**

- Prepare a page showing a number line from 0 to 10 or use the resource, B1, in the blackline master packet. Distribute a copy to each student. Ask students to use the number line to model addition expressions such as 2 + 5 and 3 + 6. Guide them to verbalize a rule that describes how to model addition on a number line. **Start at the position of the first addend, then move to the right the number of places given by the second addend.**
- Tell students that in this lesson they will use counters and number lines to model the addition of integers. Ask students to speculate how modeling addition using counters and number lines might be different when negative numbers are used.

Subtopic 1 — Adding Integers Using Counters Model

Expand Their Horizons

In Subtopic 1, students use counters to add integers. Be sure students understand what each counter represents. It may be helpful to introduce them to two-color counters before viewing the lesson. After explaining the meaning of the two different colors, ask students to model different numbers using counters. For example, have them model positive five using five yellow counters and negative two using two red counters.

Be sure students understand the term *zero pair* and the concept behind it. A zero pair is a pair of counters for which the overall value is zero. Each counter is the opposite of the other, so they "cancel each other out." Remind students that in many real-life situations actions and their opposites often "cancel each other out." For example, turning on a light and then turning it off results in no overall change in lighting; tying shoelaces then untying them results in no overall change in the state of the shoelaces. Likewise, a positive unit and a negative unit can cancel each other out. That is, they have zero value.

To ensure students understand how to form a zero pair, show different pairs of counters (some same-color, some zero pairs), and ask them to identify which are zero pairs. Finally, place a small pile of red and yellow counters on the table and ask students to identify the overall value of the counters by eliminating zero pairs.

1 To find the sum of -3 and 1, model the numbers using three red counters and one yellow counter. The yellow counter and one of the red counters form a zero pair, leaving two red counters. The two remaining red counters represent the sum, -2.

Common Error Alert:

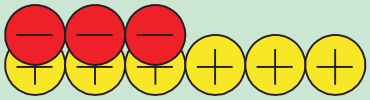
Students may see four counters in the workspace and assume the answer is 4 or -4. Remind them to focus on the color of the counters and to look for zero pairs.

2 Each addend is represented using red counters. The sum of three red counters and four red counters is seven red counters. So, -3 +(-4) = -7.

Additional Examples

1. Add using counters.
-3 + 6

Represent each addend using counters. Three zero pairs can be formed, leaving three yellow counters. The three remaining yellow counters represent the sum, +3.



-3 + 6 = 3

2. Add using counters.
-1 + (-4)

Represent each addend using counters. Since all the counters are red, the sum is -5.

-1 + (-4) = -5

Subtopic 2 Adding Integers Using Number Lines

Expand Their Horizons

In Subtopic 2, students add integers using a number line. Remind students that in an addition problem the numbers to be added are called *addends*. When using a number line to model addition, students should start at the location of the first addend and then move the number of spaces indicated by the second addend. Ask students to explain why they move right when adding a positive number and left when adding a negative number. **Adding a positive number represents adding a gain; whereas, adding a negative number represents adding a loss.**

Teachers may want to ask students if the Commutative Property of Addition applies to integers. Ask students to use a number line to determine if -2 + 4 is the same as 4 + (-2). They should discover that the two sums are the same.

3 Begin at two; then move four units to the left. The sum is -2.

4 Begin at negative three; then move two units to the left. The sum is -5.

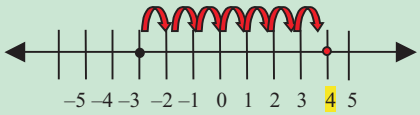
Common Error Alert:

Students may be puzzled by the use of parentheses in the addition expression -3 + (-2). Remind them that parentheses are used here to distinguish the negative sign (-) from the operation sign (+) that immediately precedes it. Students will also observe that sometimes signs will be raised or superscripted to delineate between the operations of addition and subtraction and a signed number.
Example: $^{+}2 - {}^{-}3 = {}^{-}1$

Additional Examples

1. **Add using a number line.**
   -3 + 7

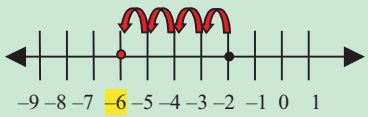
   Start at negative three. Then move right seven units.



   -3 + 7 = 4

2. **Add using a number line.**
   -2 + (-4)

   Start at negative two. Then move left four units.



   -2 + (-4) = -6

Subtopic 3 Adding Integers Using Rules

Expand Their Horizons

In Subtopic 3, students are given rules for adding integers. As they watch the DVD, use counters to reinforce and to explain the rules. When two numbers with the same sign are added, all the counters have the same color. So, the sum has the same sign as each addend. Adding numbers with different signs results in zero pairs with counters of only one color left over. (Except in the case of opposites, in which no counters are left over and the sum is zero.) So, the color of the leftover counters is the color that represents the addend with the greater absolute value.

5

The addends, -11 and 31, have different signs. Since 31 has the greater absolute value, the sum will be positive. Since 31 – 11 = 20, -11 + 31 = 20.

Common Error Alert:

Students may add the absolute values instead of subtracting. Encourage students who are having trouble with the rules to imagine how the problem could be solved using counters or a number line. Guide them in discovering that moving 31 places to the right of -11 results in landing on the number that is the difference between 31 and 11.

6

The addends are both negative, so the sum is negative. To find the sum, find |-15| + |-21|. Since |-15| + |-21| = 15 + 21 = 36, -15 + (-21) = -36.

Additional Examples

1. Add.
-14 + (-8)

Both addends are negative. The sum will be negative. Add |–14| and |–8| to find the absolute value of the sum.

-14 + -8 = -22

2. Add.
16 + (-12)

The addends have different signs. The sum will be positive, because |16| > |-12|.

16 + (-12) = 4

Look Beyond

In the next lesson, students will learn how to subtract integers. Since subtracting a number is the same as adding its opposite, students will need to have good addition skills involving integers before moving on to subtraction.

Connections

Addition can be used to represent successive changes in a measurement. When a change is represented using signed numbers, successive changes can be found by adding. For example, when a stock's value is given at the beginning of a quarter, daily changes (given as signed numbers) can be used as addends to find the stock's value at the end of several days.

Calculator

Show students how to add and subtract integers using a four-function calculator. Most of these calculators have a key for changing the sign of a number. Show them how to enter negative numbers using the +/– key. Then, show them the keystrokes necessary to find sums such as -3 + 5 = 2.

3 +/– + 5 =