Numbers and Operations

Module 3

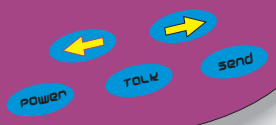
Integers

Lesson 1
Integers and Absolute Value

Teacher Notes 3.1

Objectives

- Define negative integers.
- Find the absolute value of a number.
- Graph integers on a number line.
- Compare and order integers.



Prerequisites

Adding, subtracting, multiplying, and dividing whole numbers

Comparing and ordering numbers

Vocabulary

Negative numbers
Positive numbers
Opposite numbers
Absolute value
Absolute value symbol
Integers
Infinite
Magnitude
Direction
Number line
Whole Numbers

Get Started

- Tell the students that they are going to play a personal finance game. Each student will receive an index card. Some cards will show a bank balance, representing money the student has; others will receive a statement of debt, showing the amount of money owed to creditors. Explain that each card shows the student's net (overall) worth.
- On each of several index cards, write a different net worth, such as "Bank Balance: $400," "Bank Balance: $20," "Debt: $40," and "Debt: $200." Make sure one card reads "No Debt, No Bank Balance."
- Ask students to stand, two at a time. Have each student read his or her card and then ask them to decide which student is better off. Repeat the activity, guiding them to see that a larger debt is worse than a smaller debt, a larger balance is better than a smaller balance, etc. Be sure to point out that the student with no debt and no balance is worse off than those with balances but better off than those with debts.

- Draw a number line on the board, placing the name of the student with no debt and no bank balance in the middle. Tell them that the number zero will represent this student. Ask the other students to arrange themselves in order from larger debt to smaller debt to smaller balance to larger balance and place their names on the number line in that order. Then, replace the names with the net worth on their index cards.
- Point out that this scenario necessitates the use of positive and negative numbers to distinguish between balances and debts. Many situations are similar. Tell them that in this lesson they will learn how to write, graph, compare, and order positive and negative numbers.

Subtopic 1 Negative Numbers

Expand Their Horizons

In Subtopic 1, students are introduced to the idea of negative numbers.

Ask students to imagine an elementary subtraction scenario in which a farmer has five apples. Make up simple addition and subtraction scenarios, such as *The farmer picks four more apples. How many apples does he have now?* Show them how to add apples on a number line. Then, ask them to suppose that another farmer has three apples but owes a shopkeeper eight apples. Ask them to consider what number might represent the farmer's balance of apples. Lead them to see that a negative number represents an amount less than zero. When subtracting eight from three, students must move eight places to the left of three. This requires going to the left of zero, thus resulting in a negative number.

1. The key phrase "below sea level" indicates a negative number, since sea level is considered "0" elevation. The number -2,000 represents 2,000 feet below sea level.

2. The key word "loss" indicates a negative number. Remind students that in football, the line of scrimmage can be considered "0." A gain of yards is represented using a positive number; a loss is represented using a negative number. So, negative eight represents a loss of eight yards.

3. For mountains that lie above the surface of the earth, the elevation of the mountain is described using a positive number. The number +1,705 represents the mountain's elevation.

Common Error Alert:

Students may read "-5" as "minus five" and "+4" as "plus four." Encourage them to form the habit of using the correct terms. They should read "-5" as "negative five" and "+4" as "positive four."

Additional Examples

1. **Which number can represent a gain of $15?**
   The key word “gain” indicates that a positive number should be used. The number is +15.

2. **Which number can represent 14° below zero?**
   The key word “below” indicates that a negative number should be used. The number is -14.

Subtopic 2 Opposite and Absolute Value

Expand Their Horizons

In Subtopic 2, students are introduced to opposite numbers, absolute value, and integers.

The idea of opposite numbers should come naturally to students since they lie the same distance from zero on *opposite sides* of zero. Share real-life applications of opposites (e.g. four feet above the water and four feet below the water) and ask students to consider what the opposite numbers have in common. Point out that the change from zero (e.g. water-level) is the same (four feet), but the direction is different. A number line resource sheet, B1, is provided in the blackline master packet.

This subtopic also introduces students to the absolute value of a number. Throughout this subtopic, continually remind students to think of absolute value as a measurement of distance. Ask them to read equations like |-5| = 5 not only by saying “the absolute value of negative five is five” but also by saying “the distance negative five is from zero is five.”

Integers (… -3, -2, -1, 0, 1, 2, 3 …) include whole numbers and their opposites. Remind students that the whole numbers include counting numbers and zero. Integers less than zero are negative integers. Integers greater than zero are positive integers. Zero is neither positive nor negative. It is common practice not to write the + sign with positive integers. It is simply understood that an integer with no sign is a positive integer.

The set of integers is infinite, extending without end, to the left and to the right of zero on a number line. Three dots are written to indicate that a set of integers continues without end. Integers have a magnitude, telling how much and a direction, positive or negative. On a number line, positive is to the right, and negative is to the left.

4 Negative three lies three units to the left of zero. Its opposite, three, lies three units to the right of zero. The opposite of nine is negative nine.

5 Negative twenty-three lies 23 units from zero. So, the absolute value of -23 is 23. Likewise, 64 lies 64 units from zero, so the absolute value of 64 is 64.

Additional Examples

1. What is the opposite of each number?

5 21 -1

To find the opposite of a number, find the number that lies the same distance from zero in the opposite direction.

The opposite of 5 is -5.
The opposite of 21 is -21.
The opposite of -1 is 1.

2. What is |-16| and |12|?

To find the absolute value of a number, find its distance from zero.

|-16| = 16
|12| = 12

Subtopic 3 Compare Integers

Expand Their Horizons

In Subtopic 3, students compare integers. It may be helpful to review the symbols < and > before viewing this subtopic. Remind students that the open side of the inequality symbols opens toward the greater number. Also, point out that any inequality can be written using either one of the two symbols. For example, 0 > –1 can be written -1 < 0. (In the exercises of this lesson, it is intended that the student write an inequality that shows the numbers in the given order.)

Encourage students to use logical thinking when comparing integers. Ask them to identify and verbalize rules such as *a positive number is always greater than a negative number* and *a negative number is always less than zero*.

6 On a number line, negative six lies two units to the left of negative four. So, -6 < -4.

Common Error Alert:

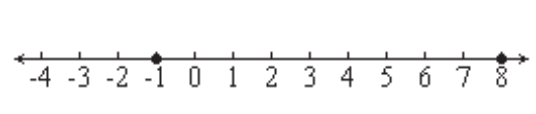
Students may compare the absolute values of the numbers instead of the numbers themselves. For example, they may write -6 > -4 because 6 > 4. Remind students to think of the numbers on a number line before comparing.

7 On a number line, negative two lies to the left of three. Therefore, -2 < 3.

Additional Examples

1. Compare -1 and 8. Write > or <.

Place each number on a number line.



Since negative one lies to the left of eight, $-1 < 8$.

2. Compare 0 and -4. Write > or <.

Place each number on a number line.

Since zero lies to the right of negative four, $0 > -4$.

Subtopic 4 Order Integers

Expand Their Horizons

In Subtopic 4, students order integers by placing each one on a number line. As they gain proficiency at comparing and ordering integers, some students may choose not to draw the number line but instead to make a mental image. Visual learners may prefer to draw the number line. Students may also need to be reminded that when finding a distance from zero, they need to count spaces between numbers as the units away from zero. Sometimes students start at zero and count the numbers instead of the spaces. For example, -3 may incorrectly be counted as four units away from zero if they count 0, -1, -2, -3 and not the spaces between these numbers.

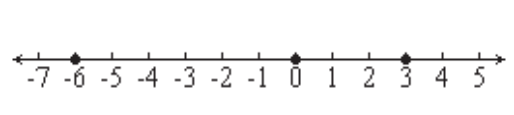
8 When plotted on a number line, the numbers appear in the order -4, -2, and 1 from left to right.

9 Place each number on a number line. In order from least to greatest, the numbers are -5, -3, -1, 0, and 2.

Additional Examples

1. Order 3, -6, and 0 from least to greatest.

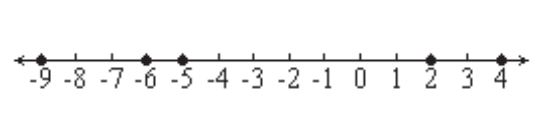
Place each number on a number line, then list the numbers as they appear from left to right.



-6, 0, 3

2. Order -5, 2, 4, -6, and -9 from least to greatest.

Place each number on a number line, then list the numbers as they appear from left to right.



-9, -6, -5, 2, 4

Look Beyond

The branch of mathematics known as Number Theory classifies numbers by their properties. *Integers* are one such classification. Students may be familiar with other classifications from the elementary (e.g. even numbers, odd numbers) to the more sophisticated (e.g. natural numbers, rational numbers, real numbers). The distinctions among the classifications are important in higher mathematics when stating definitions and theorems. A theorem that holds true for one class of numbers may not hold true for another class.

Connections

Signed numbers are used extensively in real-world applications, from temperatures above and below zero degrees to elevations above and below sea level. One of the most common uses of signed numbers occurs in the daily reports of stock and fund performance. Show the class the financial pages of a daily newspaper, pointing out that gains and losses of individual stocks are reported using signed numbers.