Numbers and Operations

★ Module 2 ★

Whole Number Operations

Lesson 5
Problem-solving Strategies

Teacher Notes 2.5

Objective

- Develop and apply a variety of strategies to solve problems; with emphasis on multi-step and non-routine problems.

power talk send

Prerequisites

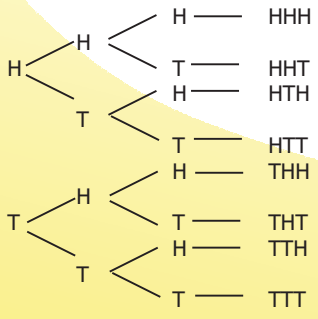
Adding, subtracting, multiplying, and dividing multi-digit whole numbers

Vocabulary

Problem-solving strategies
Venn diagram
Palindromic numbers
Tree diagram
Odd Number

Get Started

- Ask the students to find out how many possible combinations may be made by tossing a coin three times.
- Show students how to create a tree diagram to find all the outcomes using H for heads and T for tails.



- Have students use the tree diagram to find how many outcomes consist of at least two coins showing heads. **4**

- Explain to students that drawing a diagram is just one of many problem-solving strategies.

Subtopics 1 & 2 — Problem-Solving Basics and Draw a Diagram

Expand Their Horizons

In Subtopics 1 and 2, students solve problems by using the problem-solving strategy *Draw a Diagram*. They are taught to create a diagram using the information provided in the problem and to use the diagram to deduce what the problem is asking. Commonly used diagrams include the Venn diagram, the tree diagram, and geometric diagrams for area and perimeter. Even though the Venn diagram is used several times in the DVD lesson, the actual term is not presented on screen. Since the Venn is used frequently throughout the print material, the definition and description of this particular diagram might be beneficial to students. Venn diagram: a pictorial way of representing relationships between sets.

Common Error Alert:

Students may assume that they have to draw a specific kind of diagram, particularly if they have seen the teacher do several examples of one type of diagram. For example, students may use a Venn diagram when it does not apply to their problem because the teacher demonstrated a problem utilizing a Venn diagram. Also, students may approach a problem from different directions, and when they notice that their diagram is different from those made by other students around them, they may believe that they have made an error. It is important for students to understand that there may be more than one type of diagram that works for a specific problem.

1 This problem uses a diagram to keep track of who shook hands with whom. The diagram consists of five dots spaced out in a circle and line segments drawn between them. There are 10 unique line segments drawn in the diagram, so that demonstrates that there were 10 handshakes.

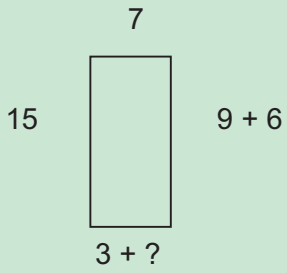
2 A Venn diagram is used to show which students are learning languages in common and which students should be separate. The diagram displays that eight students are in common, so eight is subtracted from each of the other two numbers. The total number of students is 37.

Additional Examples

1. Solve by using the problem-solving strategy *Draw a Diagram*.

Carrie was provided with five lengths of rope measuring six inches, 15 inches, nine inches, seven inches, and three inches. What is the length of a sixth piece of rope that can be used to form a rectangle?

Draw a diagram to help solve the problem. A rectangle could be drawn using the lengths of rope to find the length of the 6th piece.

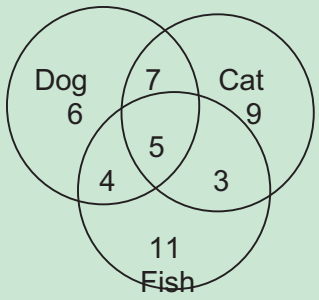


A four-inch piece of rope can be used to form a rectangle.

2. Solve by using the problem-solving strategy *Draw a Diagram*.

Betty interviewed 45 people to find out what type of pet they owned. There were 22 people who owned a dog, 24 people who owned a cat, and 23 people who owned a fish. If six people owned only a dog, three people owned both a cat and a fish, seven people owned both a dog and a fish, and five people owned all three types of pets, how many people owned both a dog and a fish? How many people owned only a cat? How many people owned only a fish?

Draw a Venn diagram that consists of three circles. Label each circle one of each type of pet. Identify the parts of the diagram that mean *only*, *both*, and *all three*. Fill in the parts of the circle with numbers in the problem that mean *only*, *both,* and *all three*. Each kind of pet will lack only one piece of its circle. Use the total number of pets owned to find the unknown parts of the circle.



Four people owned both dog and fish.
Nine people owned only a cat.
Eleven people owned only a fish.

Subtopic 3 — Look for a Pattern and Make a List

Expand Their Horizons

In Subtopic 3, students use the problem-solving strategies *Look for a Pattern* and *Make an Organized List*. Students will find possible outcomes by systematically listing all of the outcomes in a given trial or trials; they will write digits in as many ways as possible that meet a certain specification; and they will look for patterns to help them solve problems.

3 Students are asked to make an organized list of all of the possible palindromes that can be formed. They begin with the first digit and list all possibilities that begin with that digit, and then they move on to the next digit. There are nine possible palindromes.

Common Error Alert:

Students sometimes do not realize that they write the same thing down more than once in an organized list. Students must check their list and eliminate any duplicate entries.

Additional Examples

1. Solve by using the problem-solving strategy *Look for a Pattern* or *Make a List*.

Manny flipped a coin, rolled a number cube, numbered one to six, and spun a spinner that had two equal sections, black and white. How many different outcomes are possible? How many of the outcomes consist of heads, an odd number, and black section?

Make an organized list of all the possible outcomes. Count the possible outcomes. Then, count the outcomes that answer the second question.

h1b	t1b	h2b	t2b	h3b	t3b
h1w	t1w	h2w	t2w	h3w	t3w
h4b	t4b	h5b	t5b	h6b	t6b
h4w	t4w	h5w	t5w	h6w	t6w

There are 24 possible outcomes. Three outcomes consist of heads, an odd number and black section: h1b, h3b, h5b.

2. Solve by using the problem-solving strategy *Look for a Pattern* or *Make a List*.

Find the next two terms in this number pattern.

2, 5, 10, 17, ….

Examine the number pattern. Find how each number in the sequence differs from or is like the number on either side of it. Determine the rule and then find the next two terms.

Possible answers:
The pattern is the square of a number plus one.

$1^2 + 1 = 2$ $\quad$ $4^2 + 1 = 17$
$2^2 + 1 = 5$ $\quad$ $5^2 + 1 = 26$
$3^2 + 1 = 10$ $\quad$ $6^2 + 1 = 36$

OR

The differences between the numbers are $5 – 2 = 3$, $10 – 5 = 5$, $17 – 10 = 7$.
To get the next two terms add nine and then add 11.

Expand Their Horizons

In Subtopic 4, students use the problem-solving strategy *Guess and Check*. The answer is found by making an educated guess and then checking to see if the guess is accurate.

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The students are given a subtraction problem with most of the digits missing. They are given criteria for each portion of the problem and then use the strategy *Guess and Check* to solve the problem. The minuend contains only even digits and equals 80,462. The subtrahend contains only odd digits and equals 79,513. The difference is 949.

Common Error Alert:

Students sometimes get frustrated with using the *Guess and Check* strategy to solve problems. Some may not narrow their guesses down to include only reasonable guesses. Therefore, they may make wild, random guesses and as a result, greatly lengthen the time it takes to work the problem.

Additional Examples

1. **Solve by using the problem-solving strategy *Guess and Check*.**

   When 440 players attended summer football camp, the coaches divided the players into equal groups. The five extra players formed a separate group. How many groups were there and how many players were in each group?

   Set up a division problem with 440 as the dividend. Guess and check the divisor until one is found that divides into 440 with a remainder of five, or subtract the five first and look for two factors that will give a product of 435.

   Possible answer: 15 groups of 29 players or 29 groups of 15 players

2. **Solve by using the problem-solving strategy *Guess and Check*.**

   Find the missing digits in the following multiplication problem.

   $$\begin{array}{r} \square 48 \\ \times\ \square \\ \hline \square,\square 3\square \end{array}$$

   Guess the multiplier and check to see if it will produce the digits that are given in the problem. Find all the missing digits.

   Possible answer:
   $$\begin{array}{r} 348 \\ \times\ 7 \\ \hline 2{,}436 \end{array}$$

Look Beyond

Students are faced with problem solving in every mathematics course and in any course of study that requires reasoning. Students will be required to solve everyday math problems that involve basic computation and advanced math problems, which will require an assimilation of math knowledge gained from a myriad of math courses. The most successful math students are good problem solvers. A task for teachers is to instill in their students a desire to think through problems before seeking help to solve them. Tenacity in problem solving is one of the most desired qualities that a student can have.

Connections

Problem solving is one of the main goals in every mathematics course. A good problem solver will be the employee that gets promoted and who is given the most responsibilities. Students should be exposed to as many types of problems and as many types of strategies for solving problems as possible before they seek a career in any field. Valued is the employee who recognizes a problem situation, thinks through the available options, and quickly solves it.