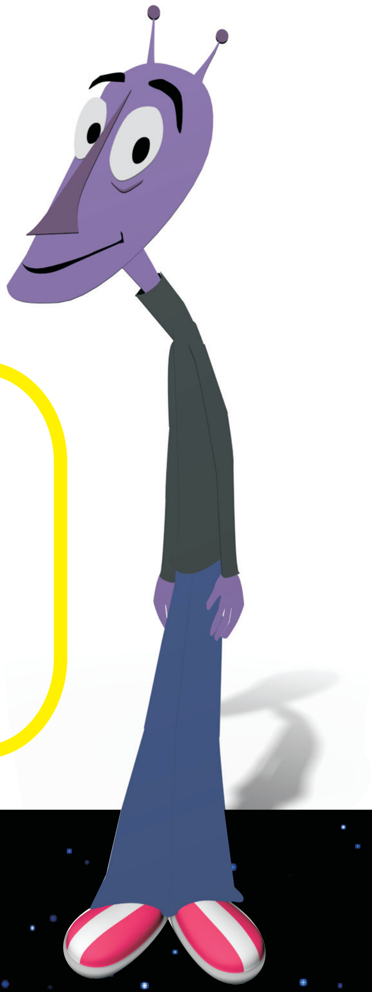
Measurement

★ Module 13 ★

Perimeter, Area, and Volume



Lesson 6
Surface Area: Pyramids and Cones

Teacher Notes 13.6

Objectives

- Derive and use formulas for surface area of pyramids and cones.
- Use square units to find the surface area of pyramids and cones.

Prerequisites

Finding the perimeter and area of a rectangle

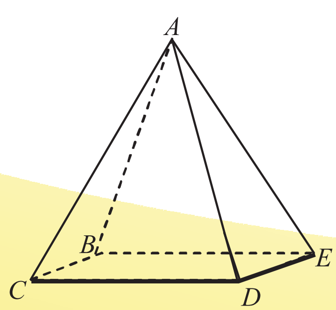
Finding the area of a circle

Vocabulary

Surface area (13.4)
Pyramid (10.4)
Net (10.5)
Lateral face (10.4)
Base (10.4)
Slant height
Area (7.1)
Perimeter (7.1)
Lateral area (13.4)
Cone (10.4)
Infinite (3.1)
Pi (9.3)
Radius (9.3)
Circumference (9.3)
Diameter (9.3)

Get Started

- As a review, ask students to name some differences between prisms and pyramids. **Prisms have two bases; pyramids have one base. In a prism, the lateral faces are parallel; in a pyramid, the lateral faces are not parallel.**
- Draw the following pyramid on the board:



- Have students name the base, the lateral faces, and the vertex.
  Base: Quadrilateral *BCDE*; Lateral faces: △*ACD*, △*ADE*, △*AEB*, △*ABC*; Vertex: *A*

- Have a volunteer draw a net of the pyramid.

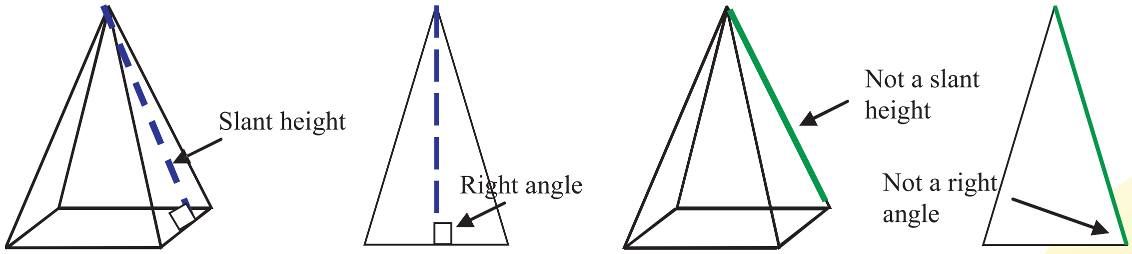
Subtopic 1 — Surface Area of a Pyramid

Expand Their Horizons

In this subtopic, students see how the formula for the surface area of a regular pyramid is derived from its net. The formula is $SA = B + \frac{1}{2}Pl$, where B is the area of the base, P is the perimeter of the base, and l is the slant height of the pyramid. The slant height of a regular pyramid is the height of a lateral face.

Common Error Alert:

Students may think that any slanted line on a pyramid is a slant height and, therefore, assume that a lateral edge is a slant height. Show students that a lateral edge is not the height of a lateral face because it is not perpendicular to the base of the triangular face.



Point out that the formula $SA = B + \frac{1}{2}Pl$ only applies to regular pyramids, that is, pyramids whose base is a regular polygon, such as a square or an equilateral triangle. Pyramids with bases that are not regular polygons do not have congruent lateral faces and, therefore, do not have a common slant height.

Since the lateral area of a solid is the sum of the lateral faces only, the formula for the lateral area of a regular pyramid is $L = \frac{1}{2}Pl$.

1 Square the length of a base edge to find the area of the base: $8^2 = 64$. To find the lateral area, find the perimeter of the base: 4(8) = 32. Next, multiply half of 32 by the slant height: 16(10) = 160. Add this to the area of the base: 160 + 64 = 224. The amount of material needed is 224 square feet.

2 Use the formula $L = \frac{1}{2}Pl$. The perimeter of the base is four times the length of one side of the base: 4(230) = 920. One-half of 920 is 460. Multiply 460 by the slant height: 460(186) = 85,560. The surface area is 85,560 square meters.

Additional Examples

1. Find the surface area of a square pyramid whose base has a perimeter of 80 square feet and whose slant height is 120 feet.

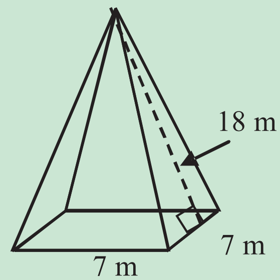
Because the base is a square, divide the perimeter by four to find the length of each side: 20 feet. The area of the base is 20^2 or 400 square feet.

Use the following formula:

$$\begin{aligned} SA &= B + \frac{1}{2}Pl \\ &= 400 + \frac{1}{2}80(120) \\ &= 400 + 4{,}800 \\ &= 5{,}200 \end{aligned}$$

The surface area is 5,200 ft^2.

2. Find the lateral area of the square pyramid.



The lateral area does not include the area of the base, so use $L = \frac{1}{2}Pl$.

Substitute 4(7) or 28 for P.
Substitute 18 for l.

$$\begin{aligned} L &= \frac{1}{2}Pl \\ &= \frac{1}{2}28(18) \\ &= 252 \end{aligned}$$

The lateral area is 252 m^2.

Subtopic 2 — Surface Area of a Cone

Expand Their Horizons

In this subtopic, students derive the formula for the surface area of a cone from the formula for the surface area of a pyramid. Recall, that as the number of sides in a regular polygon increases, the figure looks more and more like a circle. Likewise, as the number of sides in a pyramid increases, the figure looks more and more like a cone.

The formula for the surface area of a cone is $SA = \pi r^2 + \pi rl$, where r is the radius of the cone and l is the slant height. By removing the expression for the area of a circle from the formula for surface area, we get the formula for the lateral area of a cone: $L = \pi rl$.

One of the examples in the lesson involves the cone-shaped exhaust of a rocket. In the lesson, the surface area is found by substituting 3.14 for π at the same time 5.5 and 42 are substituted for *r* and *l*. Then, estimates of the base area and lateral area are summed. Show students how to estimate the surface area by first finding the exact surface area and then by substituting 3.14 for π:

$$SA = \pi r^2 + \pi r l$$
$$SA = \pi(5.5)^2 + \pi(5.5)(42)$$
$$SA = 30.25\pi + 231\pi$$
$$SA = 261.25\pi$$
$$SA \approx 261.25 \times 3.14 = 820.325$$

3 Use the formula $SA = \pi r^2 + \pi r l$, substituting four for *r* and 14 for *l*. The base has an area of 16π, and the lateral area is 56π. The surface area is 72π or about 226.08 square centimeters.

4 Because the diameter is given, first divide 72 by two to find the radius: 36 feet. The base has an area of $1{,}296\pi$, and the lateral area is $2{,}160\pi$. The surface area is $3{,}456\pi$ or about 10,852 square feet.

Additional Examples

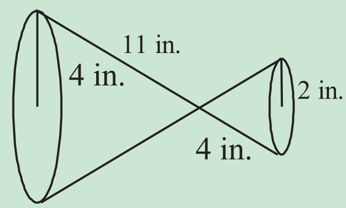
1. **Find the exact lateral area of a cone whose diameter is nine mm and whose slant height is 16 mm.**

Divide the diameter by two to find the length of the radius: 9 mm ÷ 2 = 4.5 mm. Because the base is not included, evaluate $\pi r l$ only. Do not substitute an approximation for π.

$$\pi r l = \pi(4.5)(16) = 72\pi$$

The lateral area is 72π square millimeters.

2. **Find the surface area of the figure below. The radius of each cone is shown.**



Find the surface area of each cone separately.
Larger cone: $\pi r^2 + \pi r l$

$$\pi(4)^2 + \pi(4)(11)$$
$$16\pi + 44\pi = 60\pi$$

Smaller cone: $\pi r^2 + \pi r l$

$$\pi(2)^2 + \pi(2)(4)$$
$$4\pi + 8\pi = 12\pi$$

Add the two surface areas to find the surface area of the figure:
$60\pi + 12\pi = 72\pi$, or about 226.08 sq. in.

Look Beyond

In the next lesson, students will continue their study of pyramids and cones where they will learn how to find the volume of these shapes. They will see how the formula for the volume of a pyramid is related to the volume of a prism and how the volume of a cone is related to the volume of a cylinder.

Connections

Many lighthouses are shaped like the frustum of a cone, that is, a cone with the top removed. Lighthouse keepers must know the surface area of their lighthouse to know how much paint they will need to maintain them. Lighthouses have to be painted frequently because sun exposure and saltwater quickly corrode them. Bright colors and stripes are used to help make the lighthouse stand out and serve as a guide to sailors.