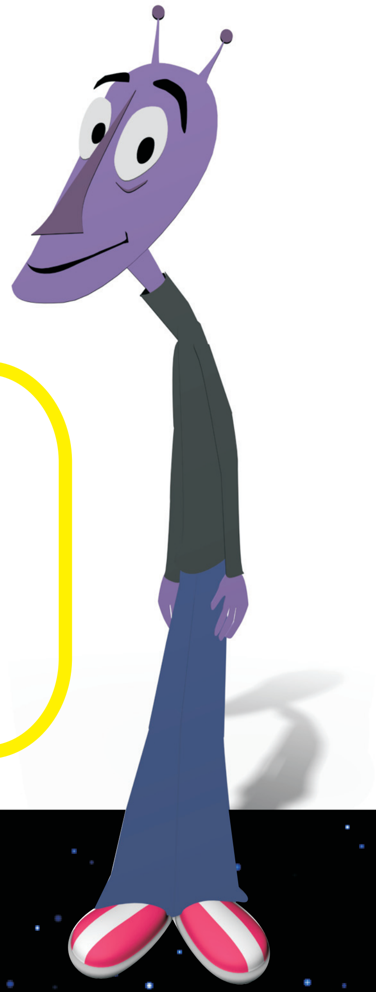
Measurement

Module 13

Perimeter, Area, and Volume



Lesson 5
Volume:
Prisms, Cylinders, and Spheres

Teacher Notes 13.5

Objectives

- Model the differences between covering the faces (surface area/nets) and filling the interior (volume).
- Derive and use formulas for the volume of prisms, cylinders, and spheres and justify using geometric models and common materials.
- Use cubic units to find the volume of prisms, cylinders, and spheres.
- Demonstrate understanding of when to use linear units to describe perimeter, square units to describe area or surface units, and cubic units to describe volume in real-world situations.
- Compare and contrast the differences among linear units, square units, and cubic units.

Prerequisites

Performing operations with whole numbers, decimals, and fractions

Evaluating expressions using the order of operations

Vocabulary

Volume (12.1)
Three-dimensional (10.4)
Surface area (13.4)
Cube (10.4)
Prism (10.4)
Edge (10.4)
Two-dimensional (9.1)
Cylinder (10.4)
Circle (9.3)
Radius (9.3)
Diameter (9.3)
Pi (9.3)
Sphere (10.4)
Perimeter (7.1)
Net (10.5)

Get Started

- Have students use unit cubes to create rectangular prisms with the following dimensions:

  a) 3 units × 5 units × 1 unit
  b) 2 units × 2 units × 2 units
  c) 4 units × 2 units × 3 units
  d) 1 unit × 7 units × 1 unit

- Have students find how many cubes were needed to build each prism.
  a) 15 cubic units, b) 8 cubic units, c) 24 square units, d) 7 square units
  Ask students if they see a relationship between the dimensions of each rectangular prism and the number of cubes needed to build the prism.
  The number of cubes is the product of the dimensions.

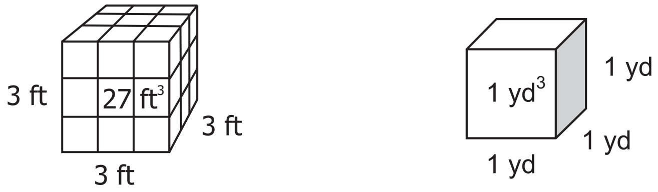
Expand Their Horizons

In this subtopic, students find the volume of cubes and other rectangular prisms. The volume of a solid, measured in cubic units, is the amount of space it contains.

The volume of a rectangular prism is found by multiplying length by width by height: $V = lwh$. The volume of a prism can also be found by using the general formula $V = Bh$, where B is the area of the base of the prism and h is the height. In the case of a rectangular prism, B is the area of a rectangle: $B = lw$. Because all of the edges of a cube are the same length, the formula for the volume of a cube can be written as $V = e^3$.

Have students use the formula for the volume of a rectangular prism to check their answers in the Get Started activity. Use this activity to reinforce that volume is measured in cubic units.

Just as in determining how many square yards are in a certain number of square feet, students often make an error in determining how many cubic yards are in a certain number of cubic feet. Draw the following diagram on the board to illustrate why there are 27 cubic feet in one cubic yard.



1 Cube the length of one edge: $3^3 = (3)(3)(3) = 27$. The volume of the block is 27 cubic centimeters.

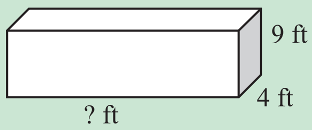
Common Error Alert:

When finding the volume of a cube, students may multiply the length of an edge by three rather than using the length as a factor three times. Remind students of the meaning of an exponent by writing a power in expanded form: $2^3 = 2 \times 2 \times 2 = 8$.

2 Multiply the three dimensions: $(8)(4)(7) = 224$. The volume of the lunchbox is 224 cubic inches.

Additional Examples

1. The volume of the box is 1,080 ft^3. What is the length of the box?



Substitute what is known into the formula for the volume of a rectangular prism. Then, solve for l.

$$V = lwh$$
$$1{,}080 = (l)(4)(9)$$
$$1{,}080 = 36l$$
$$\frac{1{,}080}{36} = l$$
$$30 = l$$

The length of the box is 30 feet.

2. Find the volume of a cube whose edge is 0.4 inch long.

Use the formula $V = e^3$.

$$V = (0.4)^3$$
$$= (0.4)(0.4)(0.4)$$
$$= 0.064$$

The volume of the cube is 0.064 cubic inches.

Subtopic 2 — Volume of a Cylinder and Sphere

Expand Their Horizons

In this subtopic, students learn the formulas for the volume of a cylinder and a sphere. As with a prism, the volume of a cylinder can be found using the formula $V = Bh$, where B is the area of the base and h is the height. In this case, B is the area of either of the circular bases of the cylinder: $B = \pi r^2$. Therefore, the volume of a cylinder can be found by using the formula $V = \pi r^2 h$.

The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere.

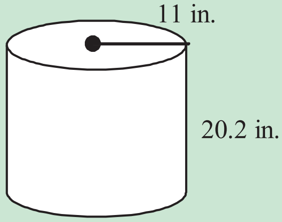
This formula can also be written as $V = \frac{4\pi r^3}{3}$.

3 Since the diameter of the canister is six inches, the radius is three inches. Use the formula $V = \pi r^2 h$. $V = \pi(3)^2(9)$. Simplify: $81\pi \approx 254.34$. The volume is about 254.34 cubic inches.

4 Substitute 29 for r in the formula for the volume of a sphere: $\frac{4}{3}\pi\ (29)^3$. The volume is about 102,109 cubic centimeters.

Additional Examples

1. **Find the exact volume.**



Using the formula, substitute 11 for *r* and 20.2 for *h*.

$V = \pi r^2 h$

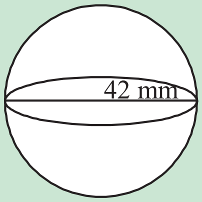
$= \pi\ (11)^2(20.2)$

$= \pi\ (121)(20.2)$

$= 2{,}444.2\pi$

The volume is $2{,}444.2\pi$ cubic inches.

2. **Find the exact volume.**



Since the diameter of the sphere is 42 millimeters, the radius is 21 millimeters.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi\ (21)^3$

$= \frac{4}{3}\pi\ (9{,}261)$

$= 12{,}348\pi$

The volume is $12{,}348\pi$ cubic millimeters.

Look Beyond

In high school geometry, students will find the volume of oblique prisms and cylinders. The prisms and cylinders in this lesson are right prisms and right cylinders. In a right prism, a lateral edge is the height of the prism. This is not true in an oblique prism. In a right cylinder, the segment joining the centers of the bases is perpendicular to the bases. In an oblique cylinder, it is not.

Connections

Landscapers and gardeners must have an understanding of volume because mulch, topsoil, fertilizer, and gravel are all sold by volume, often by the cubic foot or cubic yard. When spreading topsoil, a landscaper must not only know the length and width of the area to be covered but also the required depth of the layer of topsoil which is to be used. Buying too little topsoil will require several trips to the store to get more, while buying too much topsoil will be a waste of money.