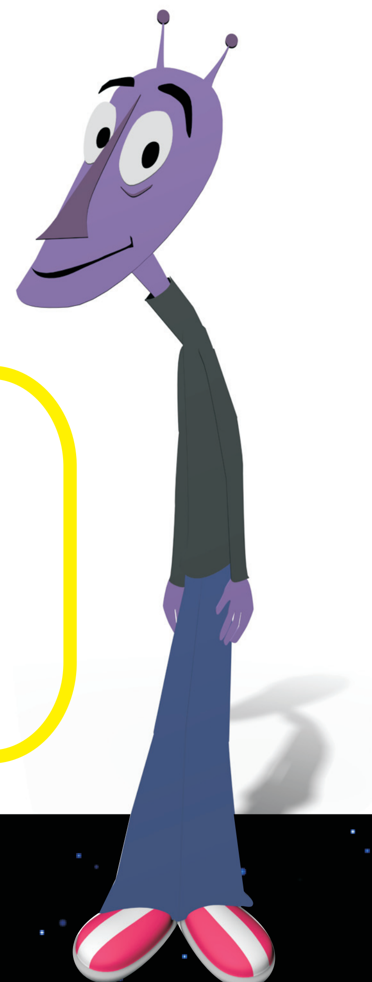
Measurement

Module 13

Perimeter, Area, and Volume



Lesson 4
Surface Area: Prisms, Cylinders, and Spheres

Teacher Notes 13.4

Objectives

- Derive and use formulas for surface area of prisms, cylinders, and spheres.
- Use square units to find the surface area of prisms, cylinders, and spheres.

Prerequisites

Finding areas of rectangles and triangles

Performing operations with whole numbers, decimals, and fractions

Evaluating expressions using the Order of Operations

Vocabulary

Surface area
Prism (10.4)
Face (10.4)
Rectangle (9.2)
Area (7.1)
Solid (10.4)
Base (10.4)
Lateral face (10.4)
Lateral area
Cylinder (10.4)
Circle (9.3)
Radius (9.3)
Circumference (9.3)
Diameter (9.3)
Pi (9.3)
Order of Operations (1.1)
Sphere (10.4)

Get Started

- Review prisms by calling on students and by having them state one fact about prisms. Continue until students run out of correct things to say.
Possible answers: The bases are congruent; the bases are parallel; a prism is named by the shape of its base; the lateral faces are rectangles; the lateral edges are parallel; the lateral edges are perpendicular to the bases; the lateral edges are congruent.

- Call on students to state facts about cylinders.
Possible answers: The bases are congruent; the bases are circles; there is one curved lateral face; the lateral surface is a rectangle when unfolded; the radius of the cylinder is the radius of a base; the height of a cylinder is the perpendicular distance between the bases.

Expand Their Horizons

In this subtopic, students find the surface area of prisms, namely rectangular and triangular prisms. As the term implies, the surface area of a figure is the area of all the surfaces that make up the figure.

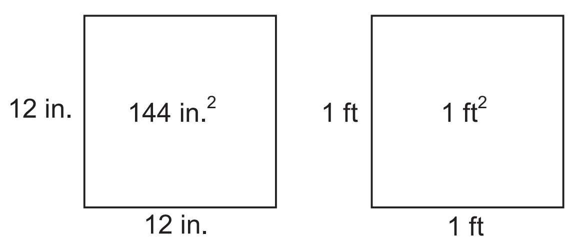
The surface area, *SA*, of a rectangular prism is the sum of the areas of its six faces. Because a rectangular prism has three pairs of congruent faces, the formula $SA = 2(lw) + 2(wh) + 2(lh)$ can be used. This formula can also be written as $SA = 2(lw + wh + lh)$.

Students do not need to memorize the formula for the surface area of a rectangular prism if they understand the meaning of surface area. They can simply find the area of each of the six rectangles that form the prism and then find the sum. Remembering that opposite faces are congruent makes this easier.

Common Error Alert:

While finding the areas of the faces of a rectangular prism, students may forget or may mix up which faces they have already found the area of and/or may not use a face at all. If a diagram of the prism is provided, have students shade in the areas of the faces as they find them. If a diagram of the prism is not provided, students can first sketch a prism and label the sides appropriately.

In the lesson example about finding the surface area of a house for Oox, the surface area is 8,496 square inches. Ask students how they could find how many square feet that would be. A common misconception is to divide by 12 because there are 12 inches in one foot. However, because this is area, divide by 12^2 or 144. Draw the following diagram on the board to illustrate why there are 144 square inches in one square foot.



To find the surface area of a triangular prism, students use the formula $SA = 2B + L$, where B is the area of one base and where L is the lateral area. The lateral area of a figure is the sum of the lateral faces. The formula $SA = 2B + L$ can be used to find the area of any prism, including rectangular prisms.

1

Find the sum of the areas of all the faces. Double the product of the length and width for the area of the top and bottom faces: 40 square inches. Double the product of the width and height for the area of the left and right faces: 96 square inches. Double the product of the length and height for the area of the front and back faces: 120 square inches. Add the areas: 256 square inches.

Additional Examples

1. Find the surface area of a rectangular prism whose length is 13 feet, whose width is seven feet, and whose height is 10 feet.

Find twice the length times the width:
2(13)(7): 182.
Find twice the length times the height:
2(13)(10): 260.
Find twice the width times the height:
2(7)(10): 140.
Find the sum of the three products:
182 + 260 + 140 = 582.

The surface area is 582 square feet.

2. Find the surface area of a cube whose side length is 0.3 inch.

Each face is a square with an area of 0.3×0.3 or 0.09 square inch.

Because the six faces are congruent, multiply by six: $0.09 \times 6 = 0.54$.

The surface area is 0.54 square inch.

Subtopic 2

Surface Area of a Cylinder

Expand Their Horizons

In this subtopic, students learn why the expression $2\pi r^2 + 2\pi rh$ gives the surface area of a cylinder, where r represents the radius of a cylinder and where h represents the height of the cylinder. The first addend, $2\pi r^2$, represents the area of both bases. The second addend in the expression, $2\pi rh$, represents the lateral, or curved, area of the cylinder

Since the diameter of a circle is twice the radius of the circle, the lateral area can be written as πdh. Therefore, if given the diameter rather than the radius of the circle, students could use the formula $SA = 2\pi r^2 + \pi dh$.

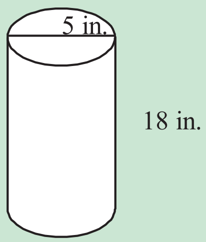
Common Error Alert:

Students may forget that without parentheses, an exponent only applies to the number or variable directly preceding it. Students who square more than the radius can write the formula as SA = $2\pi(r^2) + 2\pi rh$.

2 In the formula $SA = 2\pi r^2 + 2\pi rh$, substitute seven for r and six for h: $2\pi(7)^2 + 2\pi(7)(6)$. Simplify: $98\pi + 84\pi = 182\pi \approx 571.48$. The surface area is about 571.48 square centimeters.

Additional Examples

1. Find the surface area.

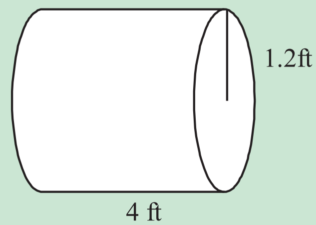


Using the formula, substitute 2.5 for r and 18 for h.

$$\begin{aligned} SA &= 2\pi rh + 2\pi r^2 \\ &= 2\pi(2.5)(18) + 2\pi(2.5)^2 \\ &= 90\pi + 12.5\pi = 102.5\pi \approx 321.85 \end{aligned}$$

The surface area is about 321.85 square inches.

2. Find the exact lateral area.



Because the lateral area is required, do not include $2\pi r^2$. Because an exact area is required, do not substitute an approximation for π.

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(1.2)(4) \\ &= 9.6\pi \end{aligned}$$

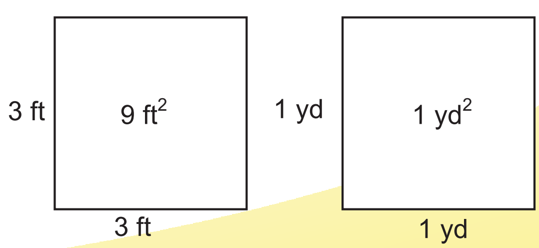
The lateral area is 9.6π square feet.

Subtopic 3 Surface Area of a Sphere

Expand Their Horizons

In this subtopic, students use the expression $4\pi r^2$ to find the surface area of a sphere.

In the lesson, students find the area of a moon model to be 4,534.16 square inches. Have students express the area in square feet and square yards. If students are uncertain about converting from square feet to square yards, draw the following:

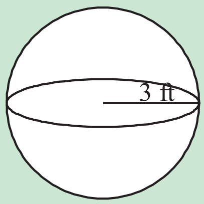


There are nine square feet in one square yard. By dividing by 144 and then by nine, students will see that 4,534.16 square inches is about 31.5 square feet or about 3.5 square yards.

3 Square the radius: $6^2 = 36$. Multiply by four: $36 \times 4 = 144$. Multiply by 3.14: $144 \times 3.14 \approx 452.16$. The surface area is about 452.16 square centimeters.

Additional Examples

1. Find the exact surface area.



Use the formula, substituting 3 for r:

$SA = 4\pi r^2 = 4\pi 3^2 = 4\pi 9 = 36\pi$

The surface area is 36π square feet.

2. The surface area of a sphere is 2,122.64 square centimeters. What is the diameter of the sphere?

Write the formula, substituting 2,122.64 for SA.

$2{,}122.64 = 4\pi r^2$
Divide both sides of the equation by four.
$530.66 = \pi r^2$
Divide both sides of the equation by π.
Use 3.14 as an approximation.
$169 \approx r^2$

The radius squared is about 169 square centimeters, so the radius is about 13 centimeters, and the diameter is twice as much: about 26 centimeters.

Look Beyond

In high school geometry, students will derive the formula for the surface area of a sphere. One common way involves imagining the sphere divided into extremely small congruent pyramids, each with its vertex at the center of the sphere and polygonal base on the surface of the sphere. By using algebra and the expression for the volume of a pyramid, the expression for the surface area can be manipulated into $4\pi r^2$.

Later in this course, students will find the surface area of pyramids and cones.

Connections

Builders of model airplanes calculate the surface area of the plane's wings when determining the wing load of the plane. Wing load is the weight of the plane divided by the surface area of the wings. If the wing load of a model plane is too high, performance, handling, and speed will suffer. The same idea holds true for real airplanes. If the wing load of a plane is too high, cargo will be removed before the plane takes off.