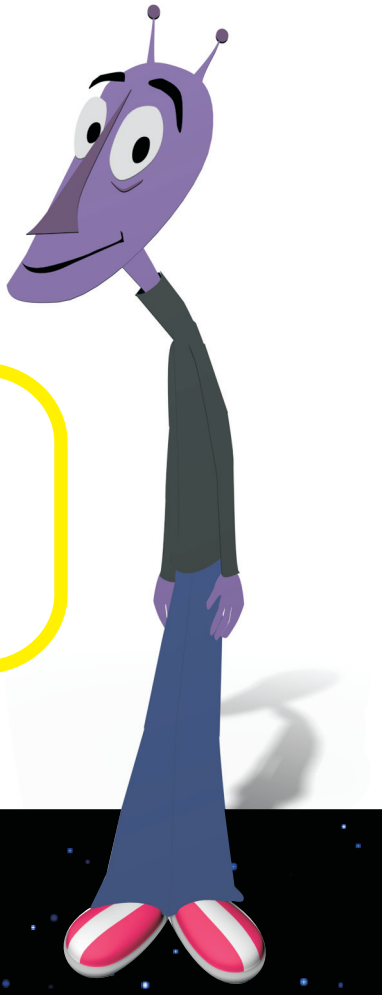
Measurement

★ Module 13 ★

Perimeter, Area, and Volume



Lesson 3
Area:
Irregular Shapes

Teacher Notes 13.3

Objectives

- Estimate and calculate the area of irregular two-dimensional shapes.
- Estimate and calculate the area of more complex or irregular two-dimensional shapes by dividing them into more basic shapes.

power talk send

Prerequisites

Finding areas of circles, parallelograms, and trapezoids

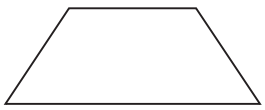
Performing operations with whole numbers, decimals, and fractions

Vocabulary

Irregular shape
Area (7.1)
Combined shape (13.3)
Pentagon (9.1)
Square (9.2)
Rectangle (9.2)
Triangle (8.4)
Circle (9.3)
Diameter (9.3)
Radius (9.3)
Pi (9.3)
Right triangle (8.4)

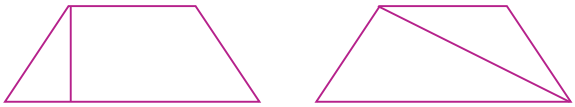
Get Started

- Draw a trapezoid on the board and ask students what shape they see.

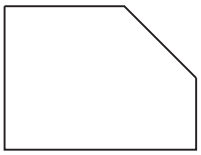


- While drawing two vertical line segments in the trapezoid as shown below, tell students that they also see one rectangle and two right triangles.

- Have students find two others ways to split up the trapezoid into other shapes they already know. **Possible answer: one right triangle and one trapezoid; two triangles**



- Draw the shape at the right on the board and have students find different ways to split it into different shapes they already know.

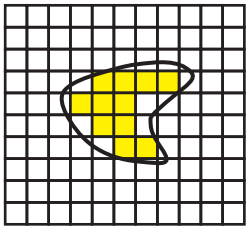


Subtopic 1 Estimating Areas of Irregular Shapes

Expand Their Horizons

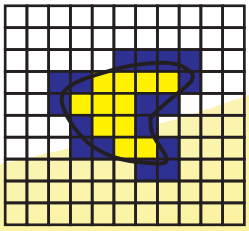
In this subtopic, students estimate the areas of irregular shapes by using a grid. Each square on the grid has a known area, such as one square mile. This type of estimating works well for finding areas of lakes, canyons, and deserts because their boundaries are often irregularly shaped and grids can easily be placed over their aerial images.

To find an estimate, first count the number of squares that are completely inside the shape. In the lesson, this number is represented by the variable *c*. In the figure below, $c = 11$.



Because the answer is an estimate, students should not stress over whether or not a square is completely inside the shape when the boundary passes very near the edge of a square.

Next, count the number of squares through which the boundary passes. This value is represented by the variable *b*. Again, students should not overly stress in determining if a square should or should not be included. In this example, $b = 17$.



To estimate the area, add half of b to c. The formula is written as $A \approx c + \frac{1}{2}b$. For the example above, the area is about $11 + \frac{1}{2}(17)$ square units, or about 19.5 square units. Because students may differ in deciding whether or not a square is "completely" inside a shape, teachers should expect to be flexible in accepting answers that are near what is given as the correct answer. Better yet, have students write out how they found their answer so teachers can determine whether or not their reasoning was correct.

For hands-on practice, copy a sheet of graph paper onto a blank transparency film. Have students draw irregular shapes on notebook size paper and use the grid to estimate the area. An alternative method would be to have students trace their hands and estimate the areas of their hands. If possible, make several transparencies, each with different size squares.

Common Error Alert:

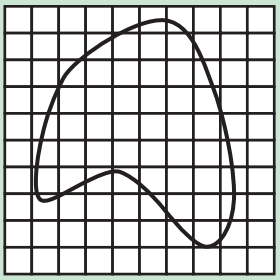
When counting, students may lose track of which squares they have already counted. Students can draw X's through the squares as they count, or they can use colored pens or pencils to distinguish between the squares inside the boundary and those on the boundary.

1 Because there are 22 squares completely inside the shape, c = 22. Because the boundary passes through 19 squares, b = 19. Add half of 19 to 22 to get 31.5 The area of the shape is about 31.5 square miles.

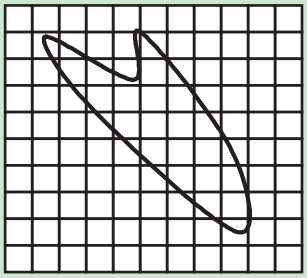
2 The value of c is 27, and the value of b is 34. Add half of 34 to 27: 27 + 17 = 44. Find the area of one grid square: one square kilometer. The area is about 44 square kilometers.

Additional Examples

1. **Estimate the area of the shape. Each ☐ is 1 m^2.**



2. **Estimate the area of the shape. Each ☐ is 1 ft^2.**



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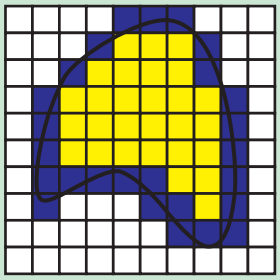
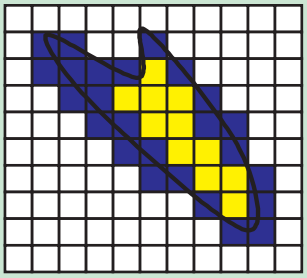
There are 28 squares inside the shape and 26 boundary squares. The area is about 28 + 13 or 41 m^2.



There are 11 squares inside the shape and 25 boundary squares. The area is about 11 + 12.5 or 23.5 m^2.

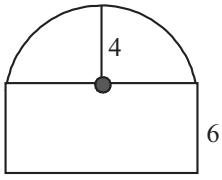


Subtopic 2 Areas of Combined Shapes

Expand Their Horizons

In this subtopic, students find areas of combined shapes and areas of shaded regions. As students saw in the Get Started activity, some shapes can be divided into two or more identifiable shapes. If students know how to find the area of the sub-divided shapes, they can find the area of the whole shape by adding the areas.

For some problems, it may appear as if there is not enough information to find some of the areas. Remind students that opposite sides of parallelograms are congruent and that all radii in a given circle are congruent. The figure below shows a rectangle and a semi-circle. The length of the rectangle is eight because it is formed by two radii, each of length four.



Common Error Alert:

When finding the area of one semi-circle, students may forget to divide their answer for the area of a circle by two. It may help if students write a formula for the whole shape before substituting values. For the figure above,

$A = \frac{\pi r^2}{2} + lw = \frac{\pi 16}{2} + 8(6) \approx 25.12 + 48 \approx 73.12$ **units**^{2}.

3 Add the area of the rectangle, 4 × 16 or 64 square inches, to the area of the triangle, $\frac{1}{2}(12)(4)$ or 24 square inches, to get an area of 88 square inches.

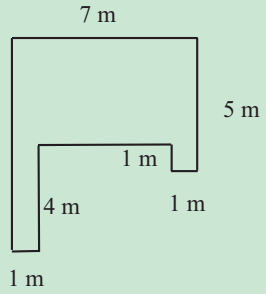
Students may notice that the whole shape is a trapezoid with base lengths of 16 and 28 inches and a height of four inches.

4 To find the area of the shaded region, find the area of the whole shape and subtract the area of the region that is not shaded. In this case, the whole shape is a triangle, and the nonshaded region is a rectangle. The area of the shaded region is $\frac{1}{2}bh - lw$ or $\frac{1}{2}(12)(15) - 9(3) = 90 - 27 = 63$ square meters.

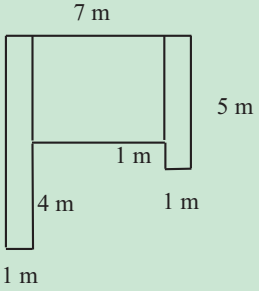
5 Subtract the area of the triangle from the area of the circle:
$\pi r^2 - \frac{1}{2}bh = \pi(6.5)^2 - \frac{1}{2}(12)(5) = 102.665 \approx 102.67$ square inches.

Additional Examples

1. Find the area of the shape.



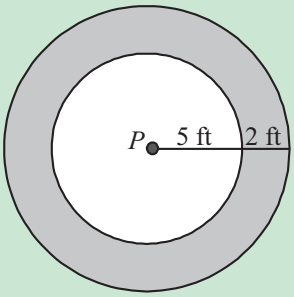
Divide the shape into three rectangles and find the area of each.



From left to right the areas are the following:
$1 \text{ m} \times 8 \text{ m} = 8 \text{ m}^2$
$5 \text{ m} \times 4 \text{ m} = 20 \text{ m}^2$
$1 \text{ m} \times 5 \text{ m} = 5 \text{ m}^2$

2. Find the area of the gray shaded region.

Circle *P*



Subtract the area of the inner circle from the area of the outer circle.

Outer circle: $A = \pi r^2 = \pi 7^2 = 49\pi \approx 153.86$

Inner circle: $A = \pi r^2 = \pi 5^2 = 25\pi \approx 78.5$

$153.86 - 78.5 = 75.36$

The area of the shaded region is about 75.36 ft^2.

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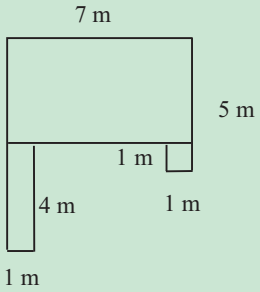
The shape can also be divided like this.



The areas are the following:
$1 \text{ m} \times 4 \text{ m} = 4 \text{ m}^2$
$7 \text{ m} \times 4 \text{ m} = 28 \text{ m}^2$
$1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2$

Either way, the combined area is 33 m^2.

Look Beyond

In high school geometry, students will find the areas of regular polygons by dividing them into congruent isosceles triangles, each with its vertex at the center of the polygon. They will use trigonometry to find the height of each triangle, which is called the apothem of the polygon.

In calculus, students will see that the area under a curve can be estimated by drawing several narrow rectangles or trapezoids under the curve and then by finding their sum.

Connections

Interior designers and homeowners must find the area of combined shapes when tiling or carpeting an area that is not a basic shape. For a sense of harmony and flow, rooms and hallways not separated by a door often have the same flooring, increasing the possibility that the area to be covered is not a simple rectangle. Moreover, many detached rooms, especially in attics and basements, have more than four walls.