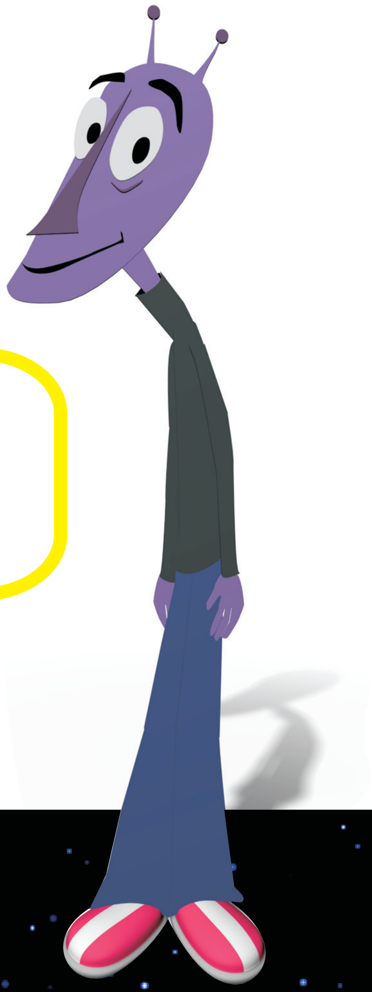
Measurement

Module 13

Perimeter, Area, and Volume



Lesson 2
Area

Teacher Notes 13.2

Objectives

- Establish and apply formulas to find the area of triangles and different types of quadrilaterals.
- Develop and use strategies to solve problems involving the area of quadrilaterals and the area of a circle.
- Demonstrate understanding of when to use linear units to describe perimeter and square units to describe area.
- Find different areas for a given perimeter and find different perimeters for a given area.

Prerequisites

Performing operations with whole numbers, decimals, and fractions

Evaluating expressions using the Order of Operations

Vocabulary

Area (7.1)
Square (9.2)
Square number (7.1)
Rectangle (9.2)
Parallelogram (9.2)
Triangle (8.4)
Trapezoid (9.2)
Circle (9.3)
Radius (9.3)
Circumference (9.3)
Pi (9.3)
Diameter (9.3)
Perimeter (7.1)

Get Started

- Give each student in the class a piece of grid paper.
- Ask students to draw rectangles with the following dimensions:

  3 units × 5 units
  10 units × 2 units
  6 units × 4 units
  1 unit × 7 units

- Ask students to find the area of each rectangle by counting the number of square units enclosed by the rectangles.
  15 square units, 20 square units, 24 square units, 7 square units

- Ask students if they see a relationship between the dimensions of a rectangle and its area.
  The area is the product of the dimensions.

Subtopic 1 Area of Rectangles and Parallelograms

Expand Their Horizons

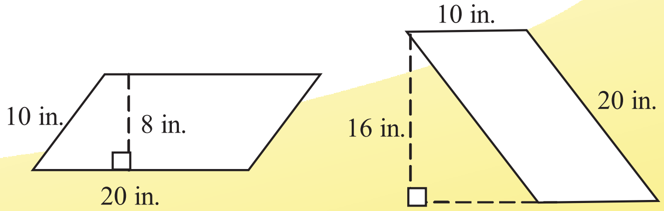
In this subtopic, students learn how to find the area of any parallelogram. They begin by reviewing the formula for the area of a square and then by becoming acquainted with the formulas for areas of rectangles and parallelograms. Point out that because squares and rectangles are also parallelograms, the formula $A = bh$ applies to those figures as well.

Have students use grid paper to draw a parallelogram (that is not a rectangle) with a base of six units and a height of three units. Have them estimate the area of the parallelogram and then use the formula to find the area. Have students draw a rectangle six units long and three units wide and find the area. Discuss why the areas of the two figures are equal. For reinforcement, students can cut the parallelogram to create a rectangle, as shown in the lesson. Teachers can also use the grid paper to reinforce the Commutative Property of Multiplication: $lw = wl$.

Common Error Alert:

Some students may use the side length of a parallelogram instead of the height when finding the area. Stress that it is the height, *h*, that is used in the formula and that height is always a perpendicular distance. In squares and rectangles, the side lengths can be used because the sides meet at right angles.

Remind students, because figures can be rotated, the base can be any side of a figure. This means that the height of the parallelogram must change accordingly. The height is the perpendicular distance between the bases. In the figure below, the same parallelogram is shown in two different orientations. If the 20-inch long segment is considered the base, then the height is eight inches. If the 10-inch long segment is considered the base, then the height is 16 inches. Either way, the area is 160 square inches.



1. Multiply the length times the width: 2.5 × 2 = 5. Because the dimensions are given in meters, the answer must be given in square meters: 5 m^2.

2. Multiply the base times the height to find the area of the parallelogram: 8 × 15 = 120. The area of the parallelogram is 120 square feet.

Additional Examples

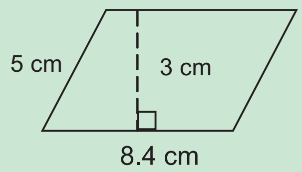
1. A rectangular driveway is 30 feet long and 12 feet wide. What is the area of the driveway?

Use the formula for the area of a rectangle.

$$
\begin{aligned}
A &= l \times w \\
&= 30 \times 12 \\
&= 360
\end{aligned}
$$

Apply the appropriate units: 360 square feet.

2. What is the area of the parallelogram?



Use the formula for the area of a parallelogram.

$$
\begin{aligned}
A &= b \times h \\
&= 8.4 \times 3 \\
&= 25.2
\end{aligned}
$$

The area of the parallelogram is 25.2 square centimeters.

Subtopic 2 — Area of Triangles, Trapezoids, and Circles

Expand Their Horizons

In this subtopic, students learn how to find the area of triangles, trapezoids, and circles. Because a triangle is exactly one-half of a parallelogram, the area of a triangle is one-half the area of a parallelogram: $A = \frac{1}{2}bh$. Remind students that multiplying by one-half is the same as dividing by two, so the formula can also be written as $A = \frac{bh}{2}$. As with a parallelogram, the height of the triangle is dependent on which base is chosen.

The formula for the area of a trapezoid can be written as either $A = \frac{1}{2}h(b_1 + b_2)$ or as $A = \frac{1}{2}(b_1 + b_2)h$. Regardless of which way it is written, students can evaluate the expression by taking one-half of the height, one-half the sum of the bases, or one-half of their product.

In the lesson, students find the area of a crater on the moon. Craters on the moon were caused by asteroids, which hit the surface of the moon billions of years ago. Though asteroids and comets continue to hit the moon today, the occurrence is much less common. Craters have been named for scientists (mainly astronomers), philosophers, and mathematicians. Examples include Copernicus Crater, Shakespeare Crater, and Euler Crater.

Common Error Alert:

When using the formula $A = \pi r^2$, students may square the product of π and the radius. Remind students because there are no parentheses, the power of two applies only to the variable *r*. Students may wish to write the formula with parentheses: $A = \pi(r^2)$.

3 Multiply the base times the height and divide by two: $90 \times 37 = 3{,}330$ and $3{,}330 \div 2 = 1{,}665$. Alternatively, take half of 90 and multiply the result by 37: $45 \times 37 = 1{,}665$. The area of the triangle is 1,665 square feet.

4 Sum the base lengths: $14 + 18 = 32$. Multiply by the height: $32 \times 13 = 416$. Divide by two: $416 \div 2 = 208$. The area of the tray is 208 square inches.

5 To find the value of the radius squared, divide the area of the circle by pi: $452.16 \div \pi \approx 144$. This is the square of the radius. Take the square root to find the radius: $\sqrt{144} = 12$. Double this length to find the diameter: $2 \times 12 = 24$. The diameter of the circle is about 24 feet.

Additional Examples

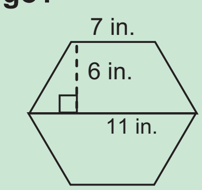
1. To the nearest centimeter, find the area of a circle whose diameter is 17 centimeters.

Use the formula $A = \pi r^2$. Find the radius by dividing the diameter by two: $17 \div 2 = 8.5$. Substitute 8.5 for *r* and 3.14 for π.

$$A = \pi r^2$$
$$A = 3.14 \times (8.5)^2$$
$$A \approx 227$$

The area of the circle is about 227 square centimeters.

2. A company logo is formed by two congruent trapezoids. What is the area of the logo?



Find the area of one trapezoid. Use the formula $A = \frac{1}{2}(b_1 + b_2)h$.

$$A = \frac{1}{2}(7 + 11)6$$
$$A = 54$$

The area of one trapezoid is 54 square inches. Since the trapezoids are congruent, the total area is $2 \times 54 = 108$ square inches.

Expand Their Horizons

In this subtopic, students look at different areas for a given perimeter of a rectangle as well as different perimeters for a given area of a rectangle. When making a table to show the different possible lengths and widths for a given perimeter, students should notice that the sum of the length and width equals half of the perimeter. Teachers can have students model the problems in the lesson by using tiles or grid paper.

When a figure is limited to a rectangular shape and the perimeter is a fixed value, the dimensions that maximize the area are those of a square. If the problem is limited to whole-number dimensions only, a square may not be possible. In that case, the dimensions with the greatest area will be those closest to forming a square. The dimensions that minimize the area are those furthest from forming a square; that is, a rectangle that is long and narrow.

With a given perimeter and no restrictions on shape, a circle will give the maximum area. To illustrate, have students find the area of a circle whose circumference is 20 yards and find the area of a square with a perimeter of 20 yards. The area of the circle is about 31.8 square yards, which is greater than the square's area of 25 square yards.

6 Find all possible whole-number factors of 36 and list the factor pairs for the length and width. Find each perimeter. The least perimeter occurs when the length and width are both six yards. With these dimensions, the perimeter is 24 yards, so 24 one-yard sections are needed.

Additional Examples

1. The area of a rectangle is 100 square feet. Using only whole-number dimensions, what are the greatest and least possible perimeters for the rectangle?

Create a table with all possible combinations of length and width for an area of 100 square feet.

Area (sq ft)	Length (ft)	Width (ft)	Perimeter (ft)
100	1	100	202
100	2	50	104
100	4	25	58
100	5	20	50
100	10	10	40

The greatest possible perimeter is 202 feet, and the least possible perimeter is 40 feet.

2. What is the greatest possible area for a rectangle whose perimeter must be 44 feet? 50 feet?

A square will always give the greatest possible area. Divide the perimeter, 44 feet, by four to find the length of each side of the square. Each side of the square will be 11 feet long. The area will be 11×11 or 121 square feet.

When the perimeter is 50 feet, the length of each side of the square is 12.5 feet, and the area is 12.5^2 or 156.25 square feet.

Look Beyond

In the next lesson, students will find areas of combined shapes by subdividing them into the basic shapes of squares, rectangles, parallelograms, and trapezoids, by finding the area of those individual shapes, and then by finding the sum of the areas. They will also learn how to estimate the areas of irregular shapes by using a grid.

In future math classes, students will use their knowledge of area to find geometric probabilities. For instance, a figure may consist of a square inside of a circle. In order to answer a question such as: *Find the probability that a point randomly chosen inside the circle is also inside the square*, students must be able to find the area of both figures.

Connections

Area is undoubtedly one of the most commonly used measurements in and around the home. For example, the amount of grass seed or fertilizer needed for a lawn is determined by the square footage of the lawn. The size of a ceiling fan needed to adequately cool a room is determined by the square footage of the room. The amount of paint needed for a room is determined by total square footage of the walls; while the amount of carpeting needed for a room is determined by the square footage of the floors.