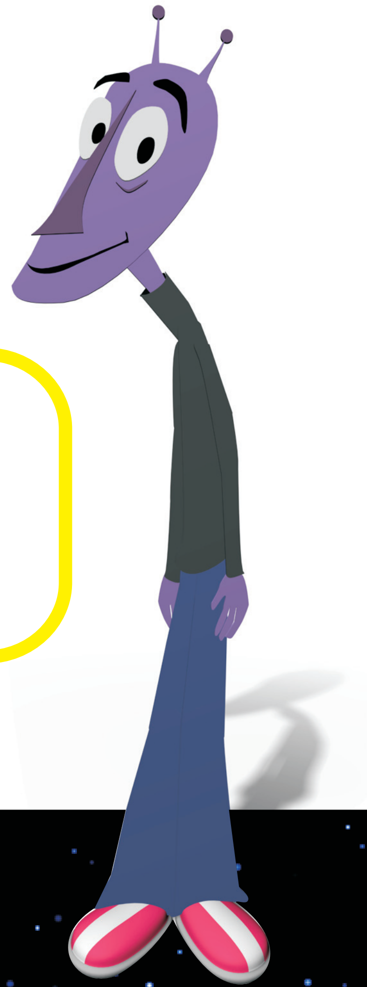
Measurement

★ Module 13 ★

Perimeter, Area, and Volume



Lesson 1
Perimeter and Circumference

Teacher Notes 13.1

Objectives

- Establish and apply formulas to find perimeter of triangles, rectangles, and parallelograms.
- Develop and use strategies to solve problems involving circumference of a circle.
- Use linear units to describe perimeter or circumference.

Vocabulary

Perimeter (7.1)
Two-dimensional (9.1)
Parallelogram (9.2)
Plane figure (9.1)
Rectangle (9.2)
Square (9.2)
Triangle (8.4)
Isosceles triangle (8.4)
Hexagon (9.1)
Circumference (9.3)
Circle (9.3)
Diameter (9.3)
Pi (9.3)
Radius (9.3)
Octagon (9.1)

Prerequisites

Evaluating expressions using the Order of Operations

Performing operations with whole numbers, decimals, and fractions

Get Started

- Give each student in the class a piece of grid paper.
- Ask students to draw a square with side lengths of five units. Then, have students find the distance around the square by counting the units. **20 units**
- Ask students to draw a rectangle with a length of six units and a width of four units. Then, have students find the distance around the rectangle by counting the units. **20 units**
- Ask students to draw another rectangle with different dimensions that has a perimeter of 20 units. **Possible answers: 7 × 3, 8 × 2, 9 × 1**
- Have students share the dimensions they used. Discuss that there are many rectangles with a perimeter of 20, but only one of them is a square.

Expand Their Horizons

In this subtopic, students are reminded that perimeter is the distance around a figure, and they learn how to find perimeters of certain shapes by using formulas. Because opposite sides of a parallelogram are congruent, the perimeter of a parallelogram can be found by using $2b + 2s$, where b represents a base and s represents a side that is not a base. Rectangles, rhombi, and squares are also parallelograms, and the same formula can be used for each. For rectangles, however, it is customary to use the terms length and width in place of base and side: $2l + 2w$. Because every side of a square has the same length, the formula simplifies to four times the length of one side or $4s$.

Common Error Alert:

Some students may struggle with deciding which sides of a parallelogram are the bases when using the formula. In general, the term *base* refers to the bottom (and top) sides of a figure, but this is relative to the figure's orientation. Show that a parallelogram can be rotated so that a side is now at the bottom and so that the formula works no matter which pair of opposite sides are chosen as the bases.

In a regular polygon, all the sides are the same length. Instead of adding all the sides, students can multiply the length of one side by the number of sides.

Common Error Alert:

Students might assume that a polygon is a regular polygon when, in fact, it is not. If a polygon is a regular polygon, it will be stated as such either in the problem or will be shown by the use of congruent slash marks in the diagram.

Stress, while formulas for perimeter can save time, they do not have to be used. Adding all the side lengths will always result in the correct answer.

1 Add the lengths of the three sides of the triangle: 12 + 12 + 4.5 = 28.5. The perimeter of the banner is 28.5 inches.

2 Substitute 150 meters and 500 meters into the formula $P = 2b + 2s$. Although the parallelogram is situated so that 150 meters is the length of the base, either value can be used as the base. The perimeter of the walkway is 1,300 meters.

Show students that the formula can also be written as $2(b + s)$ because by the Distributive Property $2(b + s) = 2b + 2s$. Therefore, an alternative is to add the base length to the side length and then double the sum to find the perimeter.

3 Since the figure is a square, multiply the side length of 22 feet by four. The perimeter of the garden is 88 feet.

Additional Examples

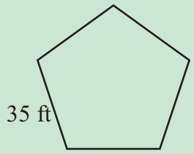
1. A rectangular porch has a perimeter of 29 feet. The width of the porch is six feet. What is the length of the porch?

Opposite sides of a rectangle are congruent. Two sides of the porch total 12 feet because 6 + 6 = 12. Subtract this from the perimeter: 29 – 12 = 17 feet.

The two remaining sides are congruent, so divide this amount by two: 17 ÷ 2 = 8.5 ft.

The porch has a length of 8.5 feet.

2. What is the perimeter of the regular pentagon?



Add the lengths of the sides.

$$P = 35 + 35 + 35 + 35 + 35$$
$$= 175$$

Another method is to multiply the length of one side by five:
5(35) = 175.

The perimeter of the pentagon is 175 feet.

Subtopic 2 Circumference

Expand Their Horizons

In this subtopic, students are reminded that the perimeter of a circle is called its circumference. Students review the formula for finding the circumference of a circle and the definition of pi, which were originally presented in Module 9 Lesson 3.

Remind students that because the circumference of a circle is its perimeter, the circumference is measured in linear units. Also, remind them π is an irrational number which never ends and never repeats. Therefore, once they substitute a value for π, the expression becomes an approximation. The decimal 3.14 and the fraction $\frac{22}{7}$ are the most common approximations for π.

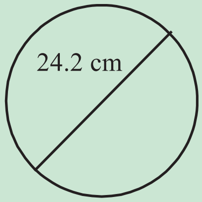
4 Use the formula $C = \pi d$ by substituting 3.14 for π and 80 for d. Rounding the answer to the nearest whole number gives a circumference of approximately 251 feet.

5 Since π times diameter equals circumference, circumference divided by π equals diameter: $\frac{C}{\pi} = d$. Divide 37.68 by π to find that the diameter is about 12 centimeters. Because the question asks for the radius and not the diameter, divide by two. The radius is about six centimeters.

In the lesson, $2r$ is first substituted for d, and $\frac{C}{2\pi} = r$. In this case, divide 37.68 by 6.28 to find that the radius is about six feet.

Additional Examples

1. The diameter of a circle is shown. Estimate the circumference of the circle to the nearest whole centimeter.



Use the formula $C = \pi d$ and substitute 24.2 for d and 3.14 for π.

$$C = \pi d$$
$$C = 3.14 \times 24.2$$
$$C \approx 76$$

The circumference of the circle is about 76 centimeters.

2. A carousel has a radius of 50 feet. The carousel makes 15 revolutions during a ride. How much distance does one horse on the carousel travel during one ride?

First find the circumference of the carousel. Since the radius is 50 feet, the diameter is 100 feet.

$$C = \pi d$$
$$C = 3.14 \times 100$$
$$C \approx 314$$

In one revolution, the horse will travel about 314 feet. Multiply this by the number of revolutions made.

$$314 \times 15 = 4{,}710$$

One horse travels about 4,710 feet during one ride.

Look Beyond

In high school geometry, students will not only find the complete distance around a circle, but they will also find the distance around part of a circle, known as an arc of the circle. To find arc length, they will use the central angle which forms the arc to determine what fraction of the circle the arc makes up and then will multiply the circumference of the circle by that amount.

Connections

When securing buildings or land areas against outside threats, authorities consider the perimeter of the building or area to determine how many security guards to post (such as one guard every 20 feet). Likewise, motion detectors can be installed on fence posts to alert when intruders are present. These sensors have a limited range, so one must know the perimeter of the enclosed area in order to know how many motion detectors to order.