Measurement

★ Module 12 ★

Attributes and Tools

Lesson 2
Same System Conversions

Teacher Notes 12.2

Objectives

- Make conversions within the same measurement system, either customary or metric, in real-world problems (e.g. hours to minutes to seconds, meters to centimeters, feet to inches, liters to milliliters, quarts to gallons, etc).
- Make conversions using time in real-world problems.

Prerequisites

Writing unit rates

Multiplying fractions and whole numbers

Writing and solving proportions

Vocabulary

Capacity (12.1)
Yardstick (12.1)
Unit rate (7.4)
Proportion (7.2)
Cross products (7.2)
Weight (12.1)
Mass (12.1)
Perimeter (7.1)
Decade
Century
Millennium
Ratio (4.1)
Leap Year

Get Started

- Write the following on the board:

  12 in. = 1 ft.

- Ask students to think about how many inches are in 2 feet. After allowing students time to think, ask them how many inches are in 2 feet. **24 in.**
- Have students share how they found their answer. **Possible answer: Two feet is one foot times two, so I multiplied 12 inches by two.**

- Show students how they could have also used the following proportion:

$$\frac{12 \text{ in.}}{1 \text{ ft}} = \frac{x}{2 \text{ ft.}}$$

- Ask students what proportion they would write to change 48 inches to feet.

$$\frac{\mathbf{12\ in.}}{\mathbf{1\ ft}} = \frac{\mathbf{48\ in.}}{\boldsymbol{x}}$$

Subtopic 1 Converting Customary Units

Expand Their Horizons

In this subtopic, students learn how to convert among customary units of length, weight, and capacity.

The lesson shows how to convert units by using unit rates. Remind students that a unit rate is a ratio with a denominator of one. For example, $\frac{3 \text{ ft}}{1 \text{ yd}}$ is a unit rate. To convert 19 yards into feet, they multiply 19 yards by the unit rate: $\frac{3 \text{ ft}}{1 \cancel{\text{yd}}} \times 19 \cancel{\text{yd}} = 57\text{ft}$. They should notice that the units of *yards* divide out, leaving the answer in feet.

Suppose they want to convert 57 feet into yards. Since the final answer is in yards, the units of *feet* need to divide out, so they should use the ratio: $\frac{1 \text{ yd}}{3 \text{ ft}}$. As written, this is not a unit rate because the denominator is not one. However, $\frac{1 \text{ yd}}{3 \text{ ft}} = \frac{\frac{1}{3} \text{ yd}}{\text{ft}}$. The last fraction is a unit rate, but not as easy to work with, so they can use the first fraction instead: $\frac{1 \text{ yd}}{3 \cancel{\text{ft}}} \times 57 \cancel{\text{ft}} = \frac{57}{3} \text{ yd} = 19 \text{ yd}$.

Note that students are shown the customary unit equivalent 2,000 lb = 1 T. This measurement is the short ton and is what is most widely used in the United States. However, this is not the only ton. The long ton is equivalent to 2,240 pounds, and the metric ton is equivalent to 1,000 kilograms, or about 2,204 pounds.

In a lesson example, Luria bought three gallons of paint, and the students determine how many pints she bought. The lesson shows how to solve this in two steps: (1) by writing a proportion, and (2) by using a unit rate. Show students that it also can be solved as one multiplication problem with three factors, as shown below.

$$3 \cancel{\text{gal}} \times \frac{4 \cancel{\text{qt}}}{1 \cancel{\text{gal}}} \times \frac{2 \text{pt}}{1 \cancel{\text{qt}}} = 24 \text{ pt}$$

The ratios are written so that the units of gallons and the units of quarts divide out and that only pints remain. By using ratios for conversions, there is no limit to how many ratios may be used at a time, allowing problems that involve several conversions to be solved in one multiplication problem.

Students may begin to notice that when they convert from a greater unit to a lesser unit, such as from yards to feet, they multiply, and when they convert from a lesser to greater unit, such as from feet to yards, they divide. Develop a line of reasoning with students. If they are converting 19 yards to feet, should the answer be more than 19 or less than 19? Why? Students should see there should be more because each yard contains three feet. Since there should be more, multiply: $19 \times 3 = 57$.

Common Error Alert:

When solving conversion problems using ratios, students often get confused as to which number goes in the numerator and which goes in the denominator. Tell students to first write the unit equivalent, such as 16 oz = 1 lb. Then, write the two possible ratios: $\frac{16\text{ oz}}{1\text{ lb}}$ or $\frac{1\text{ lb}}{16\text{ oz}}$. Then, decide which ratio would allow the units to divide out the way they want. For example, if given ounces and converting to pounds, the ounces should divide out, so the ounces need to be in the denominator: $\frac{1\text{ lb}}{16\ \cancel{\text{oz}}} \times 32\ \cancel{\text{oz}} = \frac{32}{16}\text{ lb} = 2\text{ lb}$.

1 The problem involves feet and inches, so use 12 in. = 1 ft to write two possible ratios: $\frac{12\text{ in.}}{1\text{ ft}}$ or $\frac{1\text{ ft}}{12\text{ in.}}$. To convert Ruby's height to feet, find how many inches are in four feet: $\frac{12\text{ in.}}{1\ \cancel{\text{ft}}} \times 4\ \cancel{\text{ft}} = 48\text{ in.}$ Then, add the eight inches: 48 in. + 8 in. = 56 in. Ruby's height is greater than 48 inches, so she can ride. Alternatively, students can find how many feet are in 48 inches: $\left(\frac{1\text{ ft}}{12\ \cancel{\text{in.}}} \times 48\ \cancel{\text{in.}} = 4\text{ ft}\right)$ and can see that Ruby's height is eight inches greater than the required height.

2 Because 16 oz = 1 lb and we are converting ounces to pounds, we use the ratio with ounces in the denominator so that it divides out: $\frac{1\text{ lb}}{16\ \cancel{\text{oz}}} \times 234\ \cancel{\text{oz}} = \frac{234}{16}\text{ lb} = 14\frac{10}{16}\text{ lb}$. Because there are 16 ounces in one pound, $\frac{10}{16}$ of a pound is 10 ounces, so the dog weighs 14 lb 10 oz.

3 There are two cups in one pint, so the two possible ratios are $\frac{1\text{ pt}}{2\text{ c}}$ or $\frac{2\text{ c}}{1\text{ pt}}$. We use the former ratio because we want the number of cups to divide out: $\frac{1\text{ pt}}{2\ \cancel{\text{c}}} \times 5\ \cancel{\text{c}} = \frac{5}{2}\text{ pt} = 2\frac{1}{2}\text{ pt}$. Luria drank $2\frac{1}{2}$ pt, or two pints and one cup of water.

Additional Examples

1. **Jackson is 66 inches tall. What is his height in feet and inches?**

   Write the ratio with feet and inches so that inches is in the denominator.

   $$\frac{1\text{ ft}}{12\ \cancel{\text{in.}}} \times 66\ \cancel{\text{in.}} = \frac{66}{12}\text{ ft} = 5\frac{6}{12}\text{ ft}$$

   Six-twelfths of a foot is one-half of a foot, which is six inches. Jackson's height is 5 ft 6 in.

2. **Daria's pig weighs 220 pounds. Leona's pig weighs 3,360 ounces. Which pig weighs less?**

   Convert 220 pounds to ounces, or 3,360 ounces to pounds.

   Daria's pig: $\frac{16\text{ oz}}{1\text{ lb}} \times 220\text{ lb} = 3{,}520\text{ oz}$

   Leona's pig: $\frac{1\text{ lb}}{16\text{ oz}} \times 3{,}360\text{ oz} = 210\text{ lb}$

   Leona's pig weighs less.

Subtopic 2 Converting Metric Units

Expand Their Horizons

In this subtopic, students learn how to convert among metric units of length, mass, and capacity.

The process of multiplying when changing from a greater to lesser unit and dividing when changing from a lesser to greater unit also applies to conversions in the metric system. Since metric conversions are based on the powers of 10, multiplying and dividing units can be done by simply moving the decimal point to the right or left.

Common Error Alert:

Students may have difficulty remembering which way to move the decimal point. Have these students make a table with the prefixes on top. Then, students can write the numbers they are converting below it, adding zeros as placeholders as needed. The charts below show that 5 km = 5,000 m and 5 cm = 0.05 m.

Kilo-	Hecto-	Deka-	meter gram liter	Deci-	Centi-	Milli-
5	0	0	0.			

Kilo-	Hecto-	Deka-	meter gram liter	Deci-	Centi-	Milli-
			0.	0	5	

4 Add the lengths to find the perimeter: 223 + 388 + 1,048 + 1,048 = 2,707 m. Because Julio walked the course two times, multiply by two: 2,707 × 2 = 5,414 m. To convert 5,414 m to km, move the decimal point three places to the left: 5.414 km. Alternatively, write Julio's goal in meters by moving the decimal point three places to the right: 5 km = 5,000 m. Julio met his goal.

5 To convert kilograms to grams, move the decimal point three places to the right: 5 kg = 5,000 g. This is equivalent to using a unit rate: $\frac{1{,}000\text{ g}}{1\ \cancel{\text{kg}}} \times 5\ \cancel{\text{kg}} = 5{,}000\text{ g}$.

6 To convert milliliters to liters, move the decimal point three places to the left: 4,500 mL = 4.5 L. This is equivalent to using a ratio with mL in the denominator: $\frac{1\text{ L}}{1{,}000\ \cancel{\text{mL}}} \times 4{,}500\ \cancel{\text{mL}} = \frac{4{,}500}{1{,}000}\text{ L} = 4.5\text{ L}$.

Additional Examples

1. Laurel has 7,500 mL of lemonade. She has four empty 2 L containers. Will the containers hold all of the lemonade?

To convert milliliters to liters, move the decimal point three places to the left.

7,500 mL = 7.5 L

Four empty 2 L containers will hold 4 × 2 = 8 liters. Since 8 L > 7.5 L, the containers will hold the lemonade.

2. The length of a porch is 8 m. What is the length of the porch in centimeters?

To convert meters to centimeters, move the decimal point two places to the right.

8 m = 800 cm

The porch is 800 cm long.

Subtopic 3 Converting Time Units

Expand Their Horizons

In this subtopic, students learn how to convert units of time. While most unit equivalents are constant, such as 60 sec = 1 min, the number of days in a month, and the number of days in a year vary.

There are several versions of a poem that can help students remember which months have 30 days and which have 31 days. One such version is as follows:

Thirty days have September, April, June, and November.
All the rest have 31,
Except for February which has 28 or 29.

Explain to students that the number of hours in a day is based on the Earth's rotation about its axis. It takes 24 hours for the Earth to make one full rotation; therefore, an Earth day is 24 hours long. Because each planet takes a different length of time to make one full rotation, the length of a day on each planet is different.

Likewise, every planet has a different number of days in a year because the length of a year is determined by the time it takes the planet to revolve around the sun. The lesson states that one Earth year is 365 days. This is not scientifically correct. It takes the Earth 365.25 days to revolve around the sun. To make life easier, we use 365 days every three consecutive years and 366 for the fourth. This fourth year is called a leap year. The extra day is added to February. Ask if there are any students who were born on February 29 and whether they celebrate their birthday on February 28 or March 1 for those years that do not have a February 29.

7 Students can use unit rates as before: $\frac{60 \text{ sec}}{1 \cancel{\text{min}}} \times 3 \cancel{\text{min}} = 180 \text{ seconds}$. Add the 45 seconds: 180 sec + 45 sec = 225 seconds.

Additional Examples

1. Devon bikes 30 minutes each day. How many minutes does he bike per week?

Write a ratio for 30 minutes per day and another using the fact that there are seven days in one week.

$$\frac{30 \text{ min}}{1 \cancel{\text{day}}} \times \frac{7 \cancel{\text{days}}}{1 \text{ wk}} = 210 \frac{\text{min}}{\text{wk}}$$

The units of days divide out, while the units of minute and week remain. Devon bikes 210 minutes per week.

2. On February 20, 1962, John Glenn became the first American astronaut to orbit the Earth. His flight lasted 4 hours 55 minutes 23 seconds. How long did his flight last in seconds?

Convert the hours to seconds using 60 sec = 1 min and 60 min = 1 h.

$$4 \cancel{h} \times \frac{60 \cancel{\text{min}}}{1 \cancel{h}} \times \frac{60 \text{ sec}}{1 \cancel{\text{min}}} = 14{,}400 \text{ seconds}$$

Convert the minutes into seconds.

$$\frac{60 \text{ sec}}{1 \cancel{\text{min}}} \times 55 \cancel{\text{min}} = 3{,}300 \text{ seconds}$$

Add the seconds.
14,400 + 3,300 + 23 = 17,723 seconds

Look Beyond

In later math and science courses, students will convert measurements not only within the same system but also between the systems. For instance, they will learn that one inch is equivalent to 2.54 centimeters and that one kilogram is about 2.2 pounds. The use of ratios to convert measurements is known as dimensional, or unit, analyses, and it is used heavily in high school and college chemistry and physics classes.

Connections

Many commodities are transported across the country and around the world on commercial cargo ships and cargo planes. Weight limits for these modes of transportation are usually given in tons, while the cargo loaded aboard the vessels is often weighed in pounds. The total pounds must be converted to tons to ensure that weight limits are not exceeded.