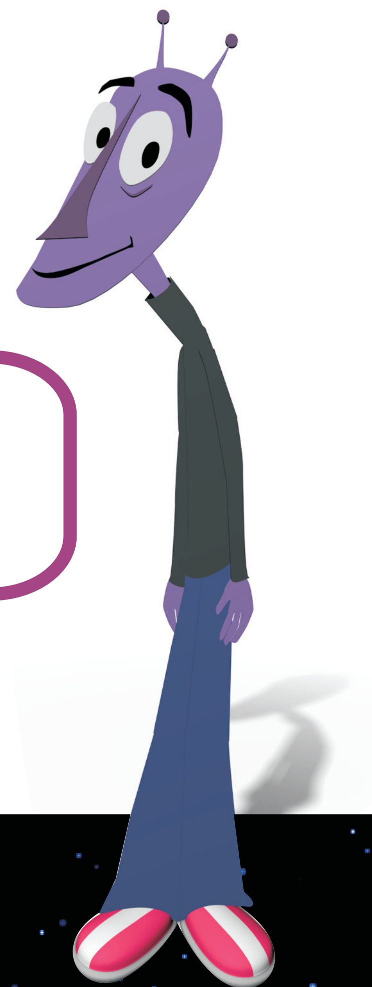
Geometry

★ Module 11 ★

Transformation of Shapes



Lesson 5
Tessellations

Teacher Notes 11.5

Objectives

- Analyze geometric patterns (e.g., tessellations, sequences of shapes) and develop descriptions of the patterns.
- Use tessellations and fractals to create geometric patterns.

Prerequisites

Finding multiples of whole numbers

Identifying regular polygons

Translating figures and rotating figures 180° about a given point

Vocabulary

Tessellation
Sequence
Term
Multiple (2.3)
Regular tessellation
Semi-regular tessellation
Midpoint (10.3)
Reflection (11.1)
Translation (11.1)
Rotation (11.2)
Transformation (11.1)

Get Started

- Write the word MATH four times in a row on the chalkboard.

  MATHMATHMATHMATH...

- Ask students what letter is in each of the following positions: first, sixth, twelfth.
  M, A, H

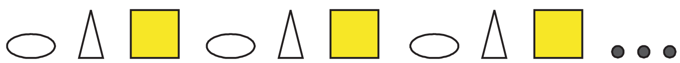
- Ask students what letter would be in each of the following positions if the pattern were continued: 17th, 18th, and 19th.
  M, A, T

- Show students that there are four letters in the pattern block, and that 17 ÷ 4 = 4R1, 18 ÷ 4 = 4R2, and 19 ÷ 4 = 4R3. Ask students how the remainders are related to which letter is in each position. **The 17th letter has a remainder of one, and the 17th letter is the same as term one; the 18th letter has remainder two, and the 18th letter is the same as term two; and the 19th letter has remainder three, and the 19th letter is the same as term three.**

Subtopic 1 Geometric Patterns

Expand Their Horizons

In this subtopic, students find missing terms in sequences by studying patterns and by writing rules for those patterns. Some of the patterns they will study will involve finding the multiple of a number. For example, in the pattern below, every multiple of three is a square. Therefore, the 18th term will be a square because 18 is a multiple of three.



Students also find missing terms in self-similar patterns. This means the figure is similar to part of itself. If the figure also has fractional dimension, it is called a fractal. The Sierpinski Triangle, which is shown in the lesson, is a popular fractal. Other popular fractals include the Koch Snowflake and the Menger Sponge, a three-dimensional fractal. Fractals can only be completed through a few steps when done on paper. However, computer programs allow us to see fractals taken through many more steps.

1 Find the number of squares in each term and look for a pattern. The pattern is to add three, so continue to add three until the seventh term is reached.

Challenge students to find a rule for the pattern. The nth term has $3n + 2$ squares.

2 Find the number of terms in each block of repeating figures. Three figures repeat. Since 31 is not a multiple of three, find the first multiple of three that is less than 31. It is 30. That means the 30th term is a triangle, and the 31st term is the figure that always follows a triangle.

Alternatively, 31 can be divided by the number of terms in the repeating block of figures and the remainder can be used: 31 ÷ 3 = 10R1. The remainder of one signifies to use the first term in the sequence.

Common Error Alert:

Students who use the latter method may confuse a remainder of zero with a remainder of one. If the remainder is zero, students should use the last term in the pattern because it means that number is a multiple of the number of terms in the repeating pattern.

3 On the next roll, the figure will look the same as the first roll, which means the number of terms in each repeating block is five. Since 38 is not a multiple of five, determine what position the figure will be in after the 35th roll. Then, count ahead three.

Additional Examples

1. What letter is in the 105th position of the repeating pattern?

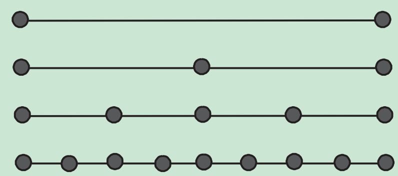
TERMTERMTERMTERM...

Four letters repeat: 105 ÷ 4 = 26R1.

Because the remainder is one, the 105th term is the same as first term, T.

Alternatively, think that the 104th term will be the letter M because 104 is a multiple of four. The 105th term will then be the next letter, T.

2. Describe the pattern below. How many circles would appear on the fifth line?



To get from one line to the one below it, draw the midpoint of every non-overlapping segment.

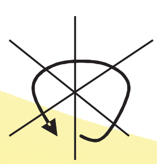
The number of consecutive circles is 2, 3, 5, and 9. In this sequence, the pattern is add one, add two, add four. To get the number of circles on the next line, double the previous number which is 8. Add to get 17.

Subtopic 2 — Tessellations

Expand Their Horizons

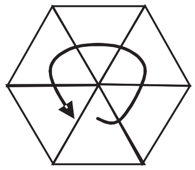
In this subtopic, students learn how to make tessellations. Most students will find this topic to be entertaining. Encourage students who find this topic intriguing to study the works of M.C. Escher, an artist who used mathematics and illusions in his drawings.

There are only three regular polygons that tessellate: the equilateral triangle, the square, and the regular hexagon. To see why, recall that the sum of the measures of the angles about a point always equals 360°.



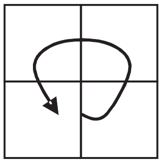
360°

In an equilateral triangle, each interior angle measures 60°. Because 360° ÷ 60° = 6, six equilateral triangles fit around a point with no gaps or overlaps.



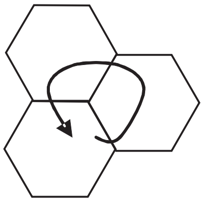
360°

In a square, each interior angle measures 90°, so four squares fit around a point with no gaps or overlaps.



360°

The interior measures of a regular hexagon measure 120°. Because 360° / 120° = 3, three hexagons fit around one point with no gaps or overlaps.



360°

Regular polygons with more than six sides will always create an overlap about a given point.

Some students will struggle with trying to draw congruent shapes when tessellating, especially as the shapes become more complex. If a copy machine is available, have students draw a tessellating shape on plain paper and then run copies of their shapes. Students can then cut out the shapes and tape them together.

4 There are two different shapes in the first tessellation, and each is a regular polygon. It is a semi-regular tessellation. The second shape is made up of rectangles. Because a rectangle is not a regular polygon, the tessellation is not a regular tessellation. The last tessellation is also made up of only one shape. Since the shape is a regular hexagon, the tessellation is a regular tessellation.

In the semi-regular tessellation, have students choose any vertex and show that the sum of the angles around the point is 360°: 4(60°) + 120° = 360°.

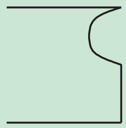
5 Rotate a copy of the figure 180° about the midpoint of a side. Copy these triangles to make the tessellation.

6 Translate and copy the figures so that there are no gaps.

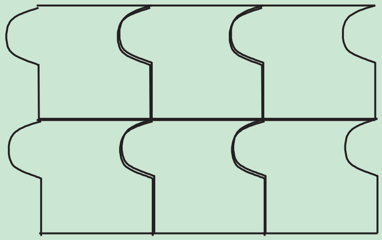
7 Rotate a copy of the figure 180° about the midpoint of a side. Copy the quadrilaterals and translate so that there are no gaps.

Additional Examples

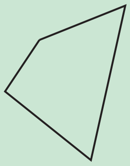
1. A square is being modified to make a tessellating shape. Finish the shape and draw the tessellation.



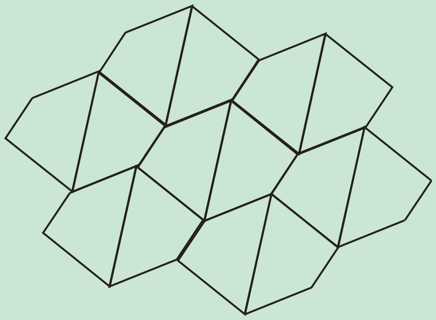
Translate a copy of the right side to the left side. Translate copies to make the tessellation.



2. Make a tessellation using the shape below.



Rotate a copy of the figure 180° and join the figures together. Copy and translate copies of these two figures to make the tessellation.



Look Beyond

Pattern recognition will become increasingly important as students progress in their mathematics classes. For now, it is enough for students to recognize and to use simple patterns. In algebra, they will write and will evaluate expressions to solve problems. They will also see patterns in function tables. They will use the patterns to determine if the function is linear, quadratic, cubic, or something else.

Connections

Patterns appear in both modern and ancient art and architecture; many of which are true tessellations, and some of which include tessellations. In Egypt, they can be found on ancient tombs and tomb walls, dresses, and utensils. In other countries, they appear on plates, ornaments, and floors. Today we see tessellations in the bricks and stones of buildings, driveways, and walkways. Tessellations also appear naturally. For example, tessellations can be seen on a honeycomb, pineapple, snakeskin, or turtle shell.