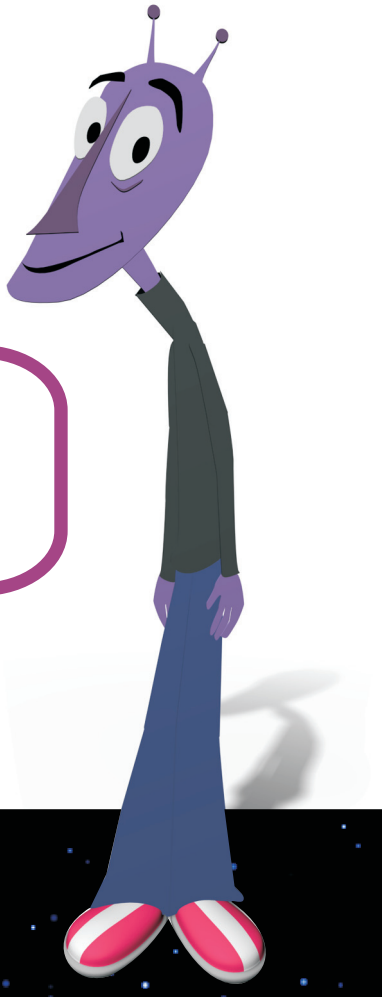
Geometry

★ Module 11 ★

Transformation of Shapes



Lesson 4
Symmetry

Teacher Notes 11.4

Objectives

- Identify lines of symmetry in two-dimensional shapes (e.g. letters of the alphabet, polygons).
- Determine if two shapes have line symmetry, rotation symmetry, and/or point symmetry.

power talk send

Prerequisites

Plotting points on the coordinate plane

Reflecting figures on the coordinate plane

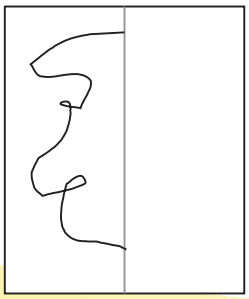
Rotating two-dimensional figures

Vocabulary

Equilateral triangle (8.4)
Line symmetry
Line of symmetry (8.5)
Reflection (11.1)
Transformation (11.1)
Reflection line
Bisect (8.5)
x-axis (10.1)
y-axis (10.1)
Rotational symmetry
Point of rotation
Order of rotation
Point symmetry

Get Started

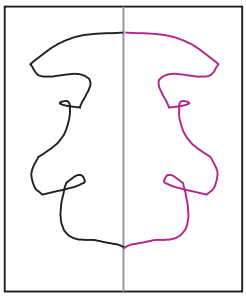
- Give each student one sheet of paper. Tell them to fold it in half lengthwise, press the fold to form a crease, and then unfold the paper.



- Have students make a design on either the left or the right side. The design must start and end on the crease and be made with one continuous line. An example is shown.

- Have students trade papers with a partner. Now tell them to make the mirror image of the design on the blank side so that, if the paper were folded, the two designs would align.

- Tell students that today's lesson is about mirror images or symmetry.



Subtopic 1 — Line Symmetry

Expand Their Horizons

In this subtopic, students learn that a figure that can be divided by a line to form two congruent mirror images has line symmetry. The division line is called a line of symmetry. Figures can have zero, one, two, three, four, or even more lines of symmetry. In fact, circles have an infinite number of lines of symmetry. Each of the lines contains a diameter of the circle.

Line symmetry is sometimes called reflectional symmetry because one side of the figure is a reflection of the other. Students are reminded that reflections flip a point or set of points across a given line. This line is a line of symmetry for the pre-image and image. On a coordinate plane, any line can be a line of symmetry, but the *x*- and *y*-axes are commonly used.

1

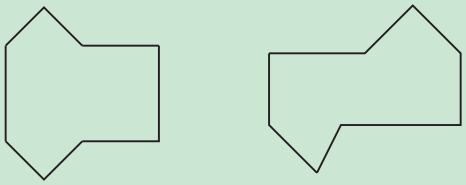
For each, draw lines so that, if the figure were folded, the two halves would be perfectly aligned. In Problem A, only a vertical line of symmetry exists. There is no horizontal line of symmetry because the top flower petals are a different length than the bottom petals. In Problem B, only a horizontal line of symmetry exists. The bottom half can be folded up to align with the top of the letter. In Problem C, a diagonal line of symmetry exists. In Problem D, the pentagon has five lines of symmetry. Each goes through a vertex and through the midpoint of the opposite side of that vertex.

2

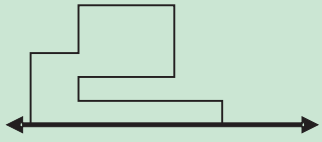
The *x*-axis is the horizontal axis. Plot the reflection of the point that is not on the *x*-axis: (0, -4). Then, draw the two remaining segments.

Additional Examples

1. **Determine which figure has line symmetry. For that figure, draw the line or lines of symmetry.**



2. **Complete the figure so it is symmetric about the given line.**



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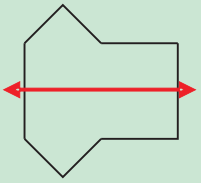
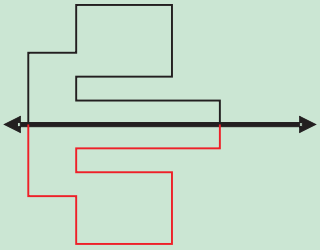
The first figure has line symmetry.



Draw the mirror image.



Subtopic 2 Rotational Symmetry

Expand Their Horizons

In this subtopic, students learn that if a figure looks like the original figure after a rotation of less than 360°, it has rotational symmetry. The definition states that the turn must be less than 360° because, for all two-dimensional figures, a 360° rotation turns the figure back onto itself.

The number of times a figure can be turned to look like itself is called the order of rotation. A square has an order of rotation of four; because during a complete turn of 360°, the figure looks like the original figure four times: at 90°, 180°, 270°, and at 360°. When listing the angles of rotation, however, we do not list 360°, so the angles of rotation for a square are said to be 90°, 180°, and 270°.

Common Error Alert:

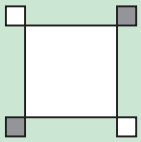
Students may use the number of angles of rotation as the order of rotation. Remind students that the order of rotation includes 360°, while the angles of rotation do not. Therefore, the order is always one greater than the number of angles.

3 Each branch of the snowflake is congruent, and they are equally spaced; so during one full turn, there are six times when the snowflake is in the same position as the original flake. This makes the order of rotation six. To find the angles of rotation, divide 360° by six: 60°. The angles of rotation are multiples of 60°, excluding 360°: 60°, 120°, 180°, 240°, and 300°.

4 There are only two times during a full turn that the figure looks like the original: halfway at 180° and after the full turn at 360°. The order of rotation is two, and the angle of rotation is 180°.

Additional Examples

1. **Name the order of rotational symmetry for the figure below.**



The figure is square, but because of the shading, the figure only looks like the original after a 180° turn and after a 360° turn. Because it happens twice, the order is two.

2. **List all the angles of rotational symmetry for the regular polygon.**

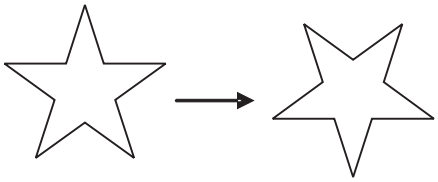
The polygon is regular, so all the sides and angles are congruent. The figure will rotate onto itself 12 times. To find the angles of rotation, divide 360° by 12: 30°. List multiples of 30°, up to and excluding 360°: 30°, 60°, 90°, 120°, 150°, 180°, 210°, 240°, 270°, 300°, and 330°.

Subtopic 3 Point Symmetry

Expand Their Horizons

In this subtopic, students learn that point symmetry is a special type of rotational symmetry. A figure has point symmetry if it has an angle of rotation of 180°. In other words, the figure looks the same after it has been rotated one half turn. Therefore, one way to check for point symmetry is to turn the figure upside down. If it looks the same as the original figure, the figure has point symmetry.

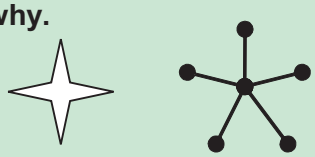
5 The figure has rotational symmetry because it can be turned less than 360° and looks like the original figure. It does not have point symmetry because180° is not one of the angles of rotation. Another way to check for point symmetry is to turn the figure upside down, as shown below. It does not look the same as the original figure.



6 The hexagon has six lines of symmetry, each dividing the figure into two congruent mirror images. Because the order of rotation is six, divide 360° by six to find how often the figure rotates onto itself. It does so every 60°, so the angles of rotation are 60°, 120°, 180°, 240°, and 300°. It has point symmetry because one of the angles is 180°.

Additional Examples

1. Tell which figure has point symmetry and why.



The first figure has point symmetry because it is in the same position after one half turn. The second figure does not because after one half-turn it looks different than the original image:

2. A figure has rotational symmetry of order 18. Does it have point symmetry? Explain how you know.

Yes, the figure has point symmetry because the angles of rotation are multiples of 20° and 9(20°) = 180°.

Look Beyond

In higher math classes, students will continue to draw transformations, including reflections on the coordinate plane. As their knowledge of algebra expands, the lines of symmetry will not only be the *x*- and *y*-axes but other lines, such as $y = x$.

In high school geometry, students will also study theorems and proofs which involve angles of rotation. They will learn, for instance, that a reflection across two intersecting lines is the same as a rotation about the point of intersection where the angle of rotation is twice the measure of the angle between the lines.

Connections

Many company logos and designs are symmetric. Some have line symmetry, some have rotational symmetry, and some have both. Graphic artists use the properties of symmetry and reflection to facilitate their design work. In drawing a design that has line symmetry, an artist may use a computer to draw one half of a figure and then use a *Reflect* command which will automatically draw the reflected image of their original figure. This ensures accuracy in their artwork and saves them time and money.