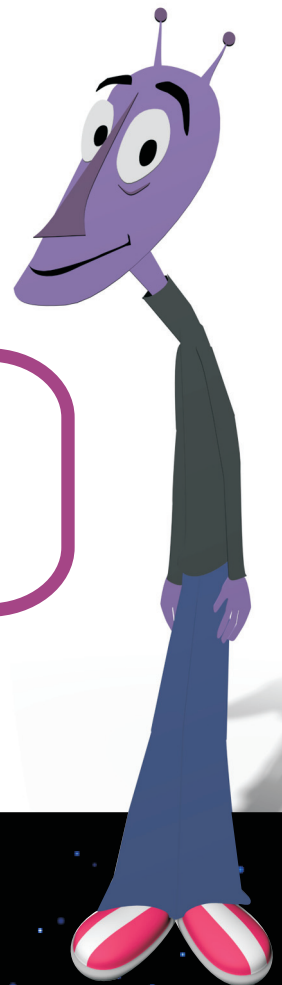


# Geometry

## ★ Module 11 ★

### Transformation of Shapes

#### Lesson 3 Dilations



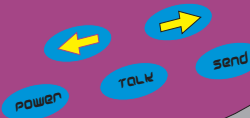


# Teacher Notes

## 11.3

### Objectives

- ♦ Draw and describe dilations (enlargements and reductions) of two-dimensional figures.
- ♦ Graph dilations on a coordinate plane.



### Prerequisites

Plotting points on the coordinate plane

Multiplying fractions and decimals

Identifying similar polygons

### Vocabulary

Dilation  
Transformation (11.1)  
Image (11.1)  
Scale factor (9.4)  
Proportion (7.2)  
Enlargement (9.4)  
Reduction (9.4)  
Center of dilation  
Coordinate plane (10.1)  
Origin (10.1)

### Get Started

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- Have students pair up with a partner. Tell each student to draw any polygon of any shape and size on a piece of paper.
- Have students trade papers with their partner. Tell students to draw the same shape they see but with dimensions that are two times larger. This is just a sketch; rulers are not required. Share some of the drawings with the entire class.
- Ask if the original and larger shapes are congruent. **No** Ask if they are similar. **Yes**

- If time permits, have students sketch the same shape as their partner's original shape, but this time, the dimensions should be half as large.
- Tell students that in today's lesson, they will learn more about enlargements and reductions and will learn what determines how the placement of the smaller or larger figure is positioned in relation to the original figure.

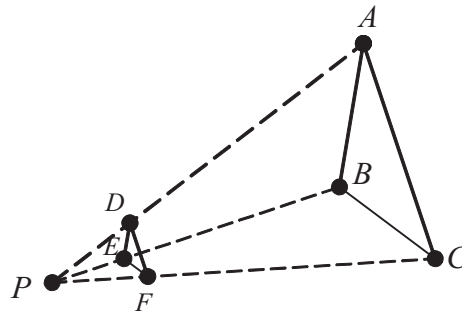
## Subtopic 1

## Dilations

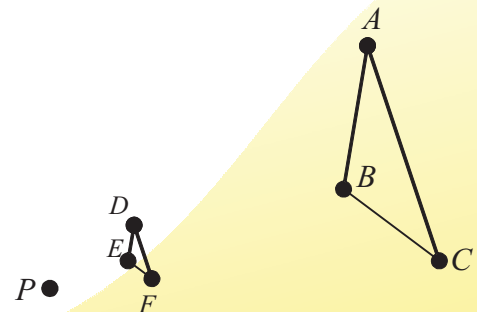
### Expand Their Horizons

In this subtopic, students draw dilations when given a scale factor and a center of dilation. In a dilation, the original and image figure are similar. They are the same shape but different in size; therefore, the corresponding sides are proportional in length.

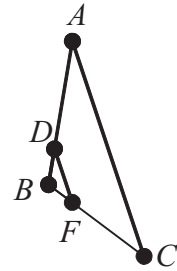
To draw a dilation, find the distance from each vertex of the original figure to the center of dilation. Multiply these distances by the scale factor. These values are the distances from the vertices of the image figure to the center of dilation. In the figure below,  $\triangle DEF$  is the image of  $\triangle ABC$  under a dilation with scale factor  $\frac{1}{4}$ . The length of segment  $PD$  is one-fourth the length of segment  $PA$ ; the length of segment  $PE$  is one-fourth the length of segment  $PB$ ; and the length of segment  $PF$  is one-fourth the length of segment  $PC$ .



The dashed lines are used to help locate the image points. They may be erased after the image is drawn. The ratio of the lengths of the corresponding sides is the scale factor. In the figure at right, the length of segment  $DE$  is one-fourth the length of segment  $AB$ . Remind students that the scale factor is used to compare side lengths and not areas. The area of  $\triangle DEF$  is *not* one-fourth of the area of  $\triangle ABC$ . It is, in fact, one-sixteenth the area of  $\triangle ABC$ .



Sometimes the center of dilation is a point on the original figure. In the figure at right,  $\triangle DBF$  is the image of  $\triangle ABC$  under a dilation with scale factor  $\frac{1}{4}$ . Point  $B$  is the center of dilation, and the length of segment  $BD$  is one-fourth the length of segment  $BA$ , and so on. When the center of dilation is a point on the original figure, that point is common to both the original and image figure.



#### Common Error Alert:

Students may use the wrong distances when determining what scale factor was used in a dilation. In the figure above, for example, students may compare the length of segment  $BD$  to the length of segment  $DA$  and say that the scale factor is  $\frac{1}{3}$ . Remind students that, as with all similar figures, the scale factor is the ratio of the corresponding sides, and segment  $DA$  is not a side of either triangle. The length of segment  $BD$  should be compared with the length of segment  $BA$ .



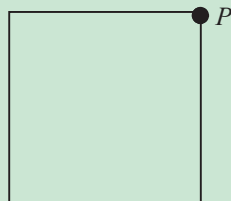
1 Measure the distance from point  $B$  to point  $A$  and divide by three. This is the length of the corresponding side on the image. Measure the distance from point  $B$  to point  $C$  and divide by three. This is the length of the corresponding side on the image. The sides of the image start at point  $B$  and extend towards the original points.



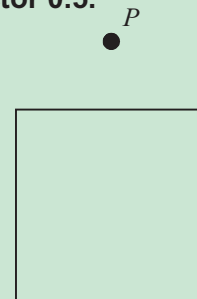
2 To find the lengths of the sides of the image figure, double the distances from  $R$  to each of the three vertices.

#### Additional Examples

1. Draw the dilation of the square with point  $P$  as the center of dilation and scale factor 0.5.

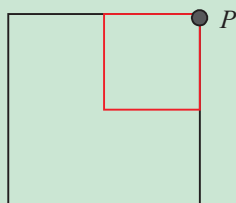


2. Draw the dilation of the square with point  $P$  as the center of dilation and scale factor 0.5.

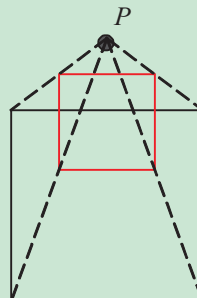


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Each side of the image square is half the length of the original square. Because point  $P$  is on the original figure, it is common to both the original figure and the image figure. Only three vertices move.



Again, each side of the image square is half the length of the original square. Because point  $P$  is not on the square, all four vertices move.



## Subtopic 2

## Dilations on the Coordinate Plane

### Expand Their Horizons

In this subtopic, students find dilations of figures on the coordinate plane. Instead of finding distances to locate image points, students can find the coordinates of image points by multiplying the coordinates of the original points by the scale factor. This method works for all dilations on the coordinate plane when the center of dilation is  $(0, 0)$ . Unless otherwise told, all dilations in the lesson have a center of dilation of  $(0, 0)$ .

When the original figure is a polygon, students find the coordinates of each vertex. If it is a segment, they find the coordinates of each endpoint. Because a circle does not have vertices, students should locate points on the circle whose coordinates are whole numbers. For instance, if a circle has a radius of two and its center is located at the origin, they can multiply the points located at  $(0, 2)$ ,  $(-2, 0)$ ,  $(0, -2)$ , and  $(2, 0)$  by two and then can sketch a circle that passes through those points.

3

Multiply each coordinate of each vertex by five.

4

Compare the coordinates of corresponding points. The coordinates of point  $X$  are  $(3, 3)$ , and the coordinates of its corresponding image point,  $S$ , are  $(1, 1)$ . The coordinates of  $S$  are one-third those of  $X$ . The same holds for all pairs of corresponding points. The scale factor is  $\frac{1}{3}$ .

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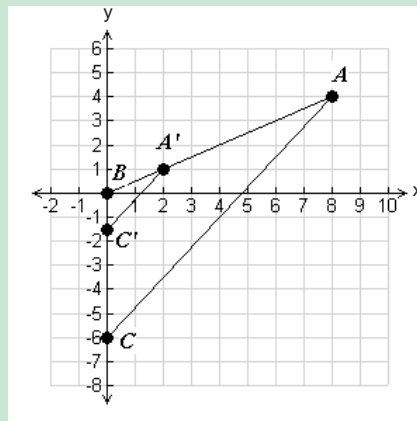
Multiply the coordinates of both endpoints by  $\frac{1}{2}$ . Plot the new coordinates and connect the endpoints to form the segment.

6

Multiply each coordinate of each vertex by two. Plot and connect the points.

### Additional Examples

1. A triangle with vertices at (3, 6), (-3, 6), and (0, -4) is dilated under a scale factor of 1.5. Give the coordinates of the image vertices.
2. What scale factor was used in the dilation below?



Multiply each coordinate by 1.5. The image vertices are located at (4.5, 9), (-4.5, 9), and (0, -6).

Compare the coordinates of an original point to an image point. Point A is located at (8, 4), and point A' is located at (2, 1). Think: *What fraction do I multiply eight by to get two, and what fraction do I multiply four by to get one?* The answer is the scale factor:  $\frac{1}{4}$ .

### Look Beyond

In future math classes, students will create dilations whose scale factor is a negative number. The image point will be the same distance away from the center of dilation as if the scale factor was its absolute value, but it will appear on the opposite ray. On a coordinate plane, the result will be a dilation and a rotation of  $180^\circ$ . Students will also expand on their knowledge of dilations to create dilations of three-dimensional figures.

### Connections

When an object is placed under a microscope, the figure appears several times larger. This is an enlargement. Common microscopes have the ability to magnify an image between 40 and 1,000 times, allowing humans to see what is otherwise difficult, or even impossible, to see with the naked eye.

