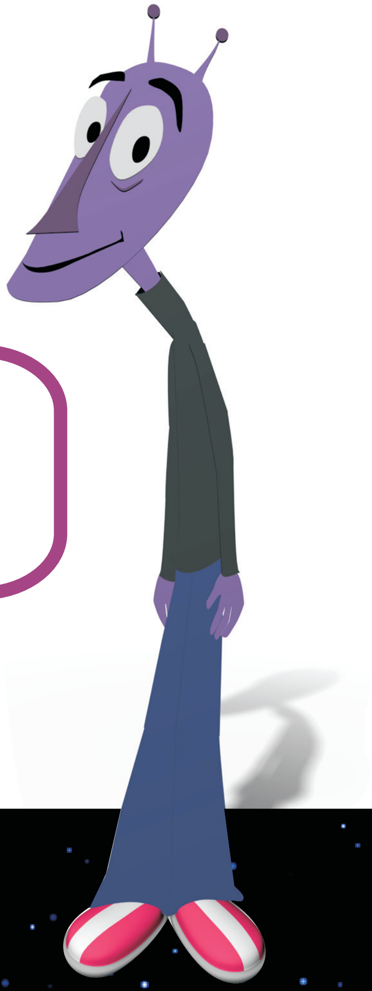
Geometry

Module 11

Transformation of Shapes



Lesson 2
Rotations

Teacher Notes 11.2

Objectives

- Perform rotations of two-dimensional figures using a variety of methods.
- Draw and describe the results of rotations about the origin (90° and 180°).

Prerequisites

Plotting points on the coordinate plane

Identifying benchmark angles of 45°, 90°, 180°, and 270°

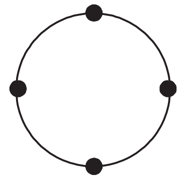
Reading and writing motion rules for transformations

Vocabulary

Rotation
Transformation (11.1)
Center of rotation
Angle of rotation
Clockwise
Counterclockwise
Equilateral triangle (8.4)
Origin (10.1)
x-coordinate (10.1)
y-coordinate (10.1)
Right angle (8.2)
Motion rule (11.1)
Ordered pair (10.1)
Trapezoid (9.2)
Quadrant (10.1)
Parallelogram (9.2)

Get Started

- Have four students stand in the front of the room and hold hands to form a circle.



- Tell students to slowly walk in their circle in the same direction until the teacher says *stop*. Say *stop* when the students have rotated 180°. The direction of the rotation does not matter.
- Ask the class how many degrees the students rotated. Have students discuss how they came up with this answer.

- Again, have students rotate until asked to *stop*. Say *stop* when the circle has rotated 90°. Ask the class how many degrees the students rotated and how they know. Repeat the process for 360°.

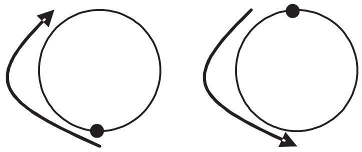
- This time, have the class say *stop* when the students rotate 45° and 270°.

Subtopic 1 Rotations of Two-Dimensional Figures

Expand Their Horizons

In this subtopic, students learn that a rotation is a transformation in which a figure is turned through a given angle about a fixed point. Figures can be turned clockwise or counterclockwise, but in this lesson, all figures are turned counterclockwise unless the directions specify otherwise.

Students may wonder why they cannot simply use the terms *left* and *right* to specify the direction of a rotation. Show that on a circle, left and right may indicate opposite directions, depending on where they start on the circle. The figures below show that *left* can indicate clockwise or counterclockwise.



Many cities in the U.S. have beltways, which are highways that run around the city, forming a somewhat circular shape. Just as left and right cannot be used above, the usual directional terms of east, west, north, and south would create the same confusion. For instance, someone entering the beltway from the north and heading west would head south until they looped around the bottom. Then, without ever changing lanes, they would be heading north again in a southbound lane! In addition, someone starting from the north and heading east would be going south but in the other lane. In other words, when starting from the north, both east and west head south. For this reason, the two lanes are usually referred to as the inner and outer loop.

Because many students have difficulty visualizing what a shape will look like as it rotates, have students cut out a shape on paper or cardboard, draw a blank set of axes on a page, set the figure on the plane, trace the figure, rotate the figure, and then trace the rotated figure. While rotating the figure with one hand, students can hold down the fixed point with the other. If using cardboard cutouts, a thumbtack can also be used. Let students experiment with cutouts of different shapes.

Teachers should stress that any point can be the center of rotation. The students should be able to see the differences this creates as they hold down different vertices of the same shape and perform the rotations. The center of rotation does not need to be a vertex. It can be any point on the shape, although the center of the shape is another commonly used point. Also, as they will see in the next subtopic, the center of rotation does not have to be on the figure at all.

Students are likely to notice that a counterclockwise rotation of 270° is the same as a clockwise rotation of 90°. Also, a rotation of 180°, in either direction, gives the same result.

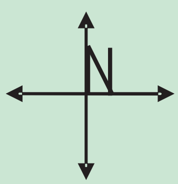
An example in the lesson shows how an equilateral triangle will rotate onto itself at 120°, at 240°, and, of course, at 360°. For any regular polygon being rotated about its center point, the figure will rotate onto itself every $\frac{360^{\circ}}{n}$, where *n* is the number of sides of the polygon. A regular pentagon will rotate onto itself every 72° because 360° ÷ 5 = 72°. This concept will be further explored in a future lesson on rotational symmetry.

1 Because it is located at the origin, the bottom center of the letter T remains fixed during the rotations.

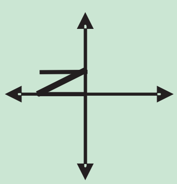
2 The vertex of the right angle remains fixed as the triangle rotates.

Additional Examples

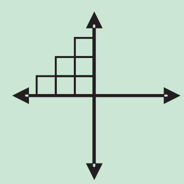
1. Rotate the figure 90° with the origin as the center of rotation.



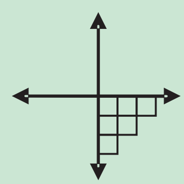
Fix the bottom left corner as the rest of the figure rotates one-quarter turn.



2. Rotate the figure 180° with the origin as the center of rotation.



The figure rotates two quarter turns.



Subtopic 2 — Rotations Using Ordered Pairs

Expand Their Horizons

In this subtopic, students learn motion rules for rotating points on a coordinate plane. The lesson begins with the point located at (3, 2) being rotated 90° and ending up at (-2, 3). Students might wonder why the rotated point is not at (-3, 2). Tell students that during a rotation, a point remains the same distance away from the center of rotation. This can be shown with their cardboard cutouts. Show that the distances have not changed by drawing the legs of the right triangles where each hypotenuse is the segment connecting each point to the origin. In both, the legs are two units and three units long, making the distances from the origin to the original and rotated points equal.

Common Error Alert:

Students may look at a motion rule such as (*a*, *b*) → (-*b*, *a*) and think that it means that (2, -4) → (-4, 2). Remind students that the sign in front of *b* means "the opposite of" and the opposite of -4 is 4, so (2, -4) → (4, 2). It may help these students to write the intermediate step: -(-4) = 4.

As with translations and reflections, polygons can be rotated by first rotating all the vertices of the polygon and then by connecting them to form the sides.

3 Use the motion rules to determine the coordinates of each rotated point.

4 Use the motion rules to determine the coordinates of the four vertices for each rotation. Then, connect the points to form the figure. The 180° rotation is the same as two consecutive reflections, one across each axis.

Additional Examples

1. **The point (6, -8) was rotated about the origin to the point (8, 6). What was the angle of rotation?**

   Look at what happened to the coordinates. The *x*- and *y*-coordinates were exchanged, and the *y*-coordinate became its opposite. This is the motion rule for a 90° rotation.

2. **The vertices of a triangle are (-5, 4), (1, 0), and (-1, -4). What are the coordinates of the triangle after a 180° rotation about the origin?**

   Take the opposite of each coordinate. Positive values become negative, and negative values become positive.

   The vertices are (5, -4), (-1, 0), and (1, 4).

Look Beyond

In future math classes, students will learn how to rotate points and figures through other angle measures, not just the benchmark angle measures used in this lesson. This can be done by using a ruler and protractor. Later in this module, they will see rotations again when they study symmetry. They will study line symmetry and rotational symmetry, where they will delve into greater depth about when and how a polygon rotates onto itself.

Connections

Skateboarders and snowboarders use rotations in describing many of the moves they make. Someone who spins around once is said to have done a 360. To do a 540, which is 360 + 180, they spin around one and one-half times. An athlete can further complicate a move by doing something else, such as grabbing their board or changing the direction of the turn. In a 180 trick for instance, the board becomes perpendicular to the ground before leveling out again.